The Development of Pre-Service Teachers' Understanding of the Knowledge Necessary to Teach Mathematics

A Case Study in Malawi

by

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“You can learn craft of painting, the choice of brushes, mixing paint, techniques, materials, colors and so on, but that does not make you a great painter. What you do with those skills is what makes you a great painter.”

David Allan Coe

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Summary

Globally, progress in education has been made in recent years to promote learning opportunities for all. However, significant gaps remain in low-income countries. In Malawi, a developing country where the educational system is under development, primary education is a major concern. Several initiatives have been made to bridge this gap through improving the quality of teacher education, but research has shown that there is still a need to further understand the learning process of pre-service teachers during the teacher education program.

The present study aimed to gain knowledge about the primary pre-service teachers' process of learning, particularly, the development of their understanding of the knowledge necessary to teach mathematics. The current study addressed the question: How do pre-service teachers develop their understanding of the knowledge necessary to teach mathematics throughout teacher education? In the study, the knowledge necessary for teaching mathematics refers to the knowledge that teachers need to carry out the tasks of teaching mathematics.

To further examine this matter, the overall question was divided into three sub-research questions: (1) What understanding do pre-service teachers have of the knowledge needed to carry out the tasks of mathematics teaching at the beginning of their teacher education? (2) To what extent does the pre-service teacher's understanding of the knowledge needed to carry out teaching tasks evolve through the discussion of practical experiences in college? (3) How do pre-service teachers develop their understanding of the knowledge necessary to carry out the tasks of teaching throughout teacher education?

These questions were compiled into a qualitative case study with six pre-service teachers in a two-year primary teacher education program at a college in Malawi. Each of these pre-service teachers represented a different profile with teaching experience, subject preferences in high school, and a favorite subject to teach during college. The research occurred over three different moments in a two-year teacher education program in which the pre-service teachers were enrolled: an initial moment at the beginning of the program consisted of a questionnaire survey and individual interviews; a second moment during teaching practice that involved mathematics lessons observations and post-lesson interviews; and a third moment at the end of the program that included a focus group discussion.

The data gathered were transcribed and analyzed using a thematic analysis approach. The themes of analysis were designed based on the six domains of mathematical knowledge for teaching theory. Findings show that pre-service teachers develop different paths of development of their understanding of the knowledge needed for teaching mathematics during teacher education and that such development has influences in how they acknowledge effective teaching in Malawi.

The current thesis includes four articles that present the main data and results of the study. The first two articles present findings from an analysis of the pre-service teachers' understanding of the subject matter knowledge and pedagogical content knowledge, and the third article presents an analysis of the pre-service teachers' understanding when discussing teaching practice. The fourth article explores the understanding pre-service teachers developed throughout the teacher education program.

The contribution of this thesis is to not only offer new empirical and theoretical insights to teacher education but also to suggest a path for further research in teaching knowledge.

Table of Contents

Acknowledgements i

Summary iii

List of Tables vii

List of Figures viii

List of Publication ix

1 Introduction 1

1.1 Research Background and Problem 1

1.2 Research Questions and Study Design 4

1.3 Articles Comprising the Thesis 5

1.4 The Researcher's Motivation Behind the Study 10

1.5 Structure of the Thesis 14

2 Theoretical Background and Relevant Research 15

2.1 Teaching Practice and Teaching Knowledge 15

2.2 Beliefs of Teaching Knowledge 24

2.3 Clarification of the Terms: Belief and Understanding 27

3 Context of the Study 33

3.1 Background: The Republic of Malawi 35

3.2 The Malawian Education System 37

3.3 Primary Teacher Education in Malawi 37

4 Methodology 45

4.1 Research Design 45

4.1.1 The Initial Moment 47

4.1.2 The Second Moment 51

4.1.3 The Third Moment 54

4.2 Ethics Issues 55

4.2.1 Acknowledgment for the Study 56

4.2.2 Issues Concerning Pre-service Teachers' Conduct 57
4.3 Data Analysis 57
5 Results 65
5.1 Article 1… 66
5.2 Article 2… 68
5.3 Article 3… 71
5.4 Article 4… 74
5.5 Summary of the Findings across Articles 77
6 Discussion 81
7 Conclusion and Implications 91
7.1 Responding to the Research Question 92
7.2 Limitations of the Study 96
7.3 Implications for Practice 97
7.4 Methodological Implications 99
7.5 Theoretical Implications 100
7.6 Further research 102
7.7 Personal Growth 103
7.7.1 As a Researcher 103
7.7.2 As a Teacher Educator 104
8 Bibliographical References 105
Appendices 133
Appendix 1 – Questionnaire Survey 133
Appendix 2 – Interview Guide 139
Appendix 3 – NSD Approval 141
Appendix 4 – Article 1 149
Appendix 5 – Article 2 161
Appendix 6 – Article 3 171
Appendix 7 – Article 4 190

List of Tables

Table 1. Overview of the thesis and research articles 8
Table 2. Core elements in numeracy and mathematics by term and number of hours (Malawian Institute of Education, 2017). 41
Table 3. Characteristics of the cases selected for the SM observations 51
Table 4. Summary of the empirical material collected in all three study moment 54
Table 5. Summary of the themes according to the cases and research questions 79

List of Figures

Figure 1. Framework of mathematical knowledge for teaching (Ball et al., 2008, p. 403)........ 21
Figure 2. Malawi and the Sub-Saharan Africa region........ 36
Figure 3. Number of participants during IM, and the selection process for the SM and TM. 48
Figure 4. Organization of the data and generation of the initial codes within the conceptual framework. 59
Figure 5. Production of themes from the initial codes and the conceptual framework. 61
Figure 6. Process of elaboration of the themes within the framework........ 61

List of Publication

Article 1

Jacinto, E. J, & Jakobsen, A. (2020). Mathematical knowledge for teaching: How do primary pre-service teachers in Malawi understand it? *African Journal of Research in Mathematics, Science and Technology Education*, *24*(1), 31─40.

Article 2

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Article 3

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Article 4

Jacinto, E. J. (2020). An analysis of pre-service teachers' understanding of the knowledge entailed in the work of teaching: Insights from Malawi. *Journal of Mathematics Teacher Education.* (Under review)

1 Introduction

1.1 Research Background and Problem

Over the last few decades, considerable advances in education and teacher education have been attained, yet they have been insufficient to promote learning opportunities for all. While statistics published by United Nations Educational, Scientific and Cultural Organization (UNESCO, 2016b) show that literacy and basic education rates have increased significantly and that the number of student enrollment and qualified teachers in schools has increased, the number of qualified teachers per student in schools remains small. Worldwide, 69 million more qualified teachers are needed to achieve the universal goals in elementary education, and unless urgent changes are made, only a few nations will be able to reach their goals in 2030 (UNESCO, 2016a). Qualified teacher shortages are particularly pronounced in Sub-Saharan African countries; a report by UNESCO (2016b) suggests these countries will require a total of 17 million primary and secondary teachers by the end of this decade.

To improve their teacher-to-student ratios, countries in Sub-Saharan Africa have been hiring candidates with suboptimal or no teaching qualifications, or persons with degrees in unrelated subjects (UNESCO, 2015a). Since the number of students who need to be taught does not match the number of qualified teachers, staff are often asked to switch departments or take over additional subjects. Professional development programs have envisioned this circumstance as an opportunity for teachers to extend their teaching expertise and maintain a high quality of teaching. However, studies have shown that these makeshift solutions might appear to be a short-term answer and are likely to have an adverse long-term impact on school practices and student outcomes (Bourdon, Frölich, & Michaelowa, 2010; Duthilleul, 2005; Farrell & Oliveira, 1993). For instance, Bourdon et al. (2010) demonstrated that, although contract teachers programs with lower qualification (lower secondary education or less) have contributed to increasing access in remote rural areas in Kenya, where regular teachers are disinclined to serve, there is a trend for significant deterioration of education quality in terms of student achievement in the long run.

Teachers are the key components of any efforts aimed at improving the educational system quality (Schleicher, 2016); thus, a lack of qualified teachers inevitably reduces instruction effectiveness and causes a decline in students' ability to learn (Darling-Hammond & Youngs, 2002; Kennedy, 2008). Without proper knowledge and skills, teachers are unable to design lessons effectively, whereby the objectives, methods, and curricular materials are well aligned and meet students' needs. Available evidence shows that a poorly elaborated lesson plan creates confusion, instability, and insecurity in the classroom, which in turn, limits students' potential to master the content (Dudek, Reddy, & Lekwa, 2019). Developing proper knowledge and skills to carry out teaching effectively is a primary task for teachers to improve the quality of education (Turner-Bisset, 2001).

As a result of this challenge, a significant amount of research has been conducted on teaching knowledge and the role such knowledge plays in the practical activities of teaching (Freire, 1985; Grossman, 1990; Shulman, 1986). Shulman (1986), particularly, made a significant contribution to this research through his studies about pedagogical content knowledge. This concept was largely based on his research aimed at elucidating what makes the learning of a specific topic easy or difficult for students, how teachers decide what to teach, where their explanations come from, and how teachers respond to students' misunderstandings. Shulman (1986) argued that during teacher education, pre-service teachers need to develop a knowledge base that includes the necessary cognitive skills, devices and values, character, and performance for creating productive learning environments. These learning environments can help pre-service teachers to increase their understanding of student interaction, allow them to explain complex concepts, adapt their teaching to individual student needs, and make the teaching process more efficient (Shulman, 1986).

In mathematics, researchers have assumed teaching knowledge is an integration of both subject matter knowledge and pedagogical content knowledge (Ball, 2017; Bruckmaier, Krauss, Blum, & Leiss, 2016; Carillo, Climent, Contreras, & Muñoz-Catalán, 2013; Rowland, 2013). As the theoretical framework for this study, Ball, Thames and Phelps (2008) Theory of Mathematical Knowledge for Teaching divided subject matter knowledge into common content knowledge (CCK), horizon content knowledge (HCK), and specialized content knowledge (SCK), whereas pedagogical content knowledge was divided into knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC). These domains provide teachers a theoretical landscape in which the knowledge necessary to teach mathematics effectively makes itself more apparent (Hill et al., 2008). This landscape is composed of teaching tasks such as capturing and retaining students' interest in learning, representing and formulating the subject to make it understandable, and the knowledge and skills necessary for carrying out these tasks effectively (Ball, 2017; Ball et al., 2008). Researchers in the field can benefit from examining these knowledge domains in practice, as gaining insight into teachers' areas of expertise could instigate new orientations for educational reforms and professional development for future teachers (Hill et al., 2008).

Although mathematics teacher knowledge is a commonly adopted term in teacher education research, there is more to teacher knowledge than its characterization. Researchers in the field have emphasized the importance of studying (and measuring) teachers' beliefs and properties of the mathematical knowledge for teaching, a crucial component to progress the quality of teaching (Delaney, Ball, Hill, Schilling, & Zopf, 2008; Fives & Buehl, 2010; Mosvold & Fauskanger, 2013, 2014). It is important to describe the type of knowledge needed for teachers; equally important is to know how pre-service teachers acquire and understand this knowledge, and how their beliefs and understandings influence the way they learn and develop as future teachers (Ball, 2017; Jakobsen, Kazima, & Kasoka, 2018; Mosvold & Fauskanger, 2013; Kasoka, Jakobsen, & Kazima, 2017).

1.2 Research Questions and Study Design

The present study contributes to the existing body of research on teacher education and teaching knowledge. In the study, teaching knowledge is defined as the knowledge teachers need to possess to carry out their work as mathematics teachers. From this definition, the overarching question guiding this study was: *How do pre-service teachers develop their understanding of the knowledge necessary to teach mathematics throughout teacher education?*

To answer this question, three sub-questions were posed:

1) What understanding do pre-service teachers have of the knowledge needed to carry out the tasks of mathematics teaching at the beginning of their teacher education?

2) To what extent does the pre-service teachers' understanding of the knowledge needed to carry out teaching tasks evolve through the discussion of practical experiences in college?

3) How do pre-service teachers develop their understanding of the knowledge needed to carry out the tasks of teaching throughout teacher education?

Answers to these sub-questions were sought through a longitudinal study involving primary pre-service teachers in Malawi, the first Sub-Saharan African country to implement free primary school education for all children in order to meet UNESCO's (2019) recommendations for the Sustainable Development Goal 4 (SDG 4) in education. The introduction of Free Primary Education was a significant step in ensuring education accessibility, but it has also created new problems for the education system and the quality of teacher education, a scenario that is further discussed in Chapter 3.

The longitudinal study described in this thesis was designed upon a case-study approach (Stake, 1995) that focused on the ways pre-service teachers in Malawi develop an understanding of the tasks of teaching mathematics and the knowledge needed to carry out these tasks in primary classrooms. The overall study was comprised of three distinct stages, each of which reflected a moment in a two-year teacher education program: an *initial moment* (IM) that occurred during pre-service teachers' theoretical courses; a *second moment* (SM) with pre-service teachers during their teaching practice in local schools; and a *third moment* (TM) at the end of teacher education when pre-service teachers come back to college for more theoretical courses. While the data collection instruments for IM consisted of a questionnaire survey and individual interviews with twenty-three pre-service teachers, those related to the SM included video records of mathematical lessons and post-lesson interviews with six of the twenty-three pre-service teachers from the IM. These six pre-service teachers presented different profiles in terms of teaching experiences, subject preference in high school, and subject preference for teaching during college. The data for the TM was obtained via a focus-group discussion with the same pre-service teachers from the SM. Further details on each of these moments as well as related data collection instruments used in this study are provided in Chapter 4 (Research Methodology).

1.3 Articles Comprising the Thesis

The data gathered as a part of the IM, SM, and TM wielded significant insights and have been published in journal articles and peer-reviewed conference papers. Four of these publications are incorporated into this thesis to build a coherent synopsis of the overall study.

In the first two articles, empirical data related to six pre-service teachers who were chosen from a twenty-three entering participant sample were examined. Analyses reported in these articles uncovered the initial understanding the pre-service teachers demonstrated of the knowledge necessary to carry out tasks of mathematics teaching at the beginning of teacher education. While the focus of the first article was on the three domains of a teachers' subject matter knowledge (CCK, HCK, and SCK), the second article's goal was to provide an examination of pre-service teachers' initial understanding of pedagogical content knowledge domains during mathematical instruction.

In the third article, pre-service teachers' understanding of the knowledge needed for carrying out tasks of mathematics teaching was examined during their teaching experiences in local schools. This focus relates to the second research question of this thesis (To what extent does the pre-service teachers' understanding of the knowledge needed to carry out teaching tasks evolve through the discussion of practical experiences in college?). Specifically, analyses and findings reported in this third article addressed the question: How do pre-service teachers understand the knowledge necessary for sequencing tasks in mathematical instruction? Sequencing tasks for teaching is a crucial component of the SCK domain, as it allows teachers to create smooth transitions between concepts, topics, and lessons. It also helps teachers become aware of the learning goals so that they can better anticipate and assist students with their difficulties and misunderstandings. In this article, a case study of two pre-service teachers (one female and one male) was presented, while drawing upon the data captured during the IM and SM study moments.

The fourth article focused on the understanding of teaching knowledge pre-service teachers develop throughout teacher education. The findings reported in this article related to the third research question (How do pre-service teachers develop their understanding of the knowledge necessary to carry out the tasks of teaching throughout teacher education?). The analyses conducted focused on two specific themes within the SCK domain: the knowledge of instructional task progression to help students solve mathematical problems and the ability to use locally available resources to create multiple representations. These two themes were developed following the analytical process adopted in the previous articles. Table 1 provides an overview of the contribution of these four publications to the overall aims of this thesis.

Table 1. Overview of the thesis and research articles

Overall Aim of the Thesis	**The aim of this study is to gain an in-depth understanding of pre-service teachers' education in Malawi and the development of pre-service teachers' understanding of teaching knowledge for mathematics instruction in primary schools.**			
Overall Research Question	How do pre-service teachers develop their understanding of the knowledge necessary to teach mathematics throughout teacher education?			
Specific Research Question	What understanding do pre-service teachers have of the knowledge needed to carry out the tasks of mathematics teaching at the beginning of their teacher education?		To what extent does the pre-service teachers' understanding of the knowledge needed to carry out teaching tasks evolve through the discussion of practical experiences in college?	How do pre-service teachers develop their understanding of the knowledge needed to carry out the tasks of teaching throughout teacher education?
Articles	Article 1	Article 2	Article 3	Article 4
Title	Mathematical knowledge for teaching: How do primary pre-service teachers in Malawi understand it?	Pedagogical content knowledge: What matters for pre-service teachers in Malawi?	Understanding of the knowledge necessary to sequence tasks in mathematical instruction: The case of Malawian pre-service teachers	An analysis of pre-service teachers' understanding of the knowledge necessary to teach mathematics: A case study in Malawi
Aim of the Study	To examine what understanding pre-service teachers have of the knowledge entailed in the work of teaching	To examine what understanding pre-service teachers have of pedagogical content knowledge for teaching mathematics	To examine how pre-service teachers develop their understanding of the knowledge necessary to sequence tasks given to students in mathematical instruction	To analyze how Malawian primary pre-service teachers develop their understanding of the knowledge needed to carry out the tasks of teaching mathematics throughout teacher education
Sample	Six Pre-service Teachers	Three Pre-service Teachers	Two Pre-service Teachers	Three Pre-service Teachers

Data	Questionnaire Survey Individual Interviews	Questionnaire Survey Individual Interviews	Questionnaire Survey Individual Interviews Practice Observations Post-Lesson Interviews	Questionnaire Survey Individual Interviews Practice Observations Post-Lesson Interviews Focus-Group Discussion
Thematic Analysis (Themes of Analysis)	CCK for Teaching and Learning; Relating Knowledge of Out-of-curriculum Content to HCK; The Importance of SCK in Interpreting Students' Errors and Capacities; SCK: Stimulating Mathematics Learning Through Different Approaches	Decision-making in teachers' KCT; Relations between KCT and KCC; Adaptations of the classroom activities from students' contributions and levels of understanding	Knowledge of how to sequence instructional tasks to foster students' learning of mathematics	Knowledge of instructional task progression to help students solve mathematical problems; Ability to use locally available resources to create multiple representations

Collectively, the four articles provide in-depth insight into the development of pre-service teachers' understanding of teaching knowledge during teacher education. A longitudinal approach is used in this thesis as a case study (Stake, 1995) for examining the evolution of the pre-service teachers' understanding of teaching knowledge as they progress through teacher education. As this research design was subject to several adaptations influenced by the pre-service teachers' experiences before and after work placement at local schools, this thesis provides a valuable contribution to existing research and theories on teaching knowledge, and may also inform teacher education policies and guidelines.

1.4 The Researcher's Motivation Behind the Study[1]

The researchers' role in and interactions with the research are critical aspects of a qualitative study. The relationship that researchers have with what is observed and how the data is analyzed should be considered (Noble & Smith, 2015). In this manner, the present research was motivated by my educational background, previous teaching experience, and research findings related to knowledge needed for teaching mathematics. Specifically, I earned a bachelor's degree in Mathematics (licentiate) and a master's degree in Mathematics and Science Education in Brazil (2011). As a university student, I faced numerous challenges regarding learning how to teach mathematics in elementary school. At the end of my educational program, graduates were expected to possess an in-depth understanding of the mathematical content, including advanced content knowledge such as Topology and Real Analysis, as well as pedagogical content such as Psychology and Theories of Teaching and Learning. In this curriculum, students were

[1] The personal pronoun "I" will be used in this section and conclusion chapter as they describe the researcher's personal motivations, the limits and contributions of this study. The rest remainder of the thesis is written in a more objective academic form.

expected to master pure mathematical content before learning educational theories and teaching practices.

Although I possessed a strong knowledge of mathematics, my knowledge about teaching and my ability to teach mathematics were basic. I can still vividly recall how, on my first day in a sixth-grade classroom, I struggled to get students' attention and motivate them to participate in the lesson. I remember telling my master's thesis supervisor that my first experience as a teacher was a complete disaster. The reassurance of this being a common experience prompted me to explore the issue further, which led me to educational research. A year later, I was assigned to work in a project about elementary school level teaching and learning activities related to triangle similarity, sponsored by the university. As a part of this project, I developed experimental mathematics activities based on a sociocultural model of education proposed by renowned authors such as Vygotsky, Leontiev, Luria, and Davydov. Working with a small number of students and with content that was of interest and relevance to them, opened new horizons on the meaning and purpose of mathematics education in elementary school (Jacinto & Cedro, 2006). After receiving my bachelor's degree, I began to work as a teacher in secondary education, adult education, indigenous education, and professional education for disadvantaged groups.

After two years of practical teaching experience, I started a master's degree in Mathematics and Science Education, which culminated with the dissertation titled "The pedagogical activity of Mathematics Teacher at the Program of Young and Adult Education (PROEJA)." The main goal of my dissertation project was to examine how the pedagogical actions of mathematics teachers satisfy the needs and aptitudes of young and adult learners. This experience taught me that, in order to provide exemplary teaching, it is vital to possess not only extensive knowledge of the content but also to provide opportunities for students to take ownership of the knowledge historically produced by humanity and share the meanings they created socially and culturally. In essence, school promotes real learning activities (Davydov, 1999).

Although my understanding of teaching and learning evolved substantially during my master's studies, the characteristics that describe a successful mathematics teacher were still unclear. Thus, after completing my master's degree in 2011, I started to develop proposals for a Ph.D. program, one of which related to an investigation of the primary cognitive skills that elementary pre-service teachers need to possess to teach mathematics. However, in 2012, I was invited to take part in an international educational cooperation program between Brazil and East Timor[2]. I accepted the offer and soon started working as a cooperating teacher (mentor teacher) in the Department of Mathematics and the Department of Primary and Elementary Teachers' Education at the National University of East Timor.

No cooperating teacher in our group was prepared for this work. Some of the tasks assigned to us were new, even for those with higher qualifications and working experience as university professors. For instance, we were required to revise and adapt the undergraduate program syllabus to the Timorese context in a useful and meaningful way; teach (mathematics, physics, biology …) disciplines in the Portuguese language, while valuing other languages from this context (Tetum, Bahasa Indonesian, Mambai …); use teaching methods that consider the previous knowledge and experiences of students, and

[2] East Timor is one of the poorest countries in Asia, raking 128 out of 187 countries worldwide on the United Nations Human Development Index (2017). East Timor was Portuguese colony until 1975, when Indonesian invasion that lasted until 1999 took place. Following East Timor's independence, from 1999 until 2006, the island was monitored by the United Nations (UN). Its first presidential elections were held 2011, which marked a significant moment in the history of East Timor. Since then, the Timorese education system, which was once based on the Indonesian educational model, underwent a complete restructuration in terms of language and culture, as well as political decisions. Several teachers and educators from Portuguese-speaking countries have since been sent to work closely with the local educators to train elementary teachers and university professors.

supervise university students in the production of monographs that would be useful for addressing local regions issues.

The challenges posed by these tasks made me appreciate the importance and value of the knowledge needed to carry out these tasks to deliver my work as required. As a young teacher in a foreign land lacking any practical experience or empirical evidence, I could only rely on speculation regarding the knowledge that is likely involved in these tasks. Thus, I developed some conjectures based on the knowledge of previous and the projected curriculum, knowledge of teaching methods that could be useful in teaching mathematics in a multilinguistic and multicultural context, knowledge of the history of mathematics locally produced in East Timor, and knowledge of the teaching methods used to teach at the university level. These initial explorations led to a realization that a better systematization of the tasks and knowledge demands for teaching mathematics in this context would result in a more efficient training program that could be adapted from the local and social needs of the Timorese context.

Such challenges in East Timor were the precursor to continuing to study teacher education in developing countries. I began to search for doctoral positions in this field in different universities around the world. At the beginning of the year 2017, the Faculty of Arts and Education of the University of Stavanger offered a Ph.D. position in mathematics education with several areas of investigation. One of the areas of most interest was the project *Improving quality and capacity of mathematics teacher education in Malawi (2013-2018)* that had a strong focus on teaching knowledge in mathematics and the development of mathematics teacher work. My application was accepted with a starting date of August 2017.

The opportunity to contribute to a study in an international cooperation program about primary teacher education in Malawi further inspired me to study the knowledge requirements for pre-service teachers to learn to teach mathematics successfully in this context. After studying the Malawian history and culture, as well as the institutional documents created for gaining mathematics teacher qualifications, which are discussed in depth in Chapter 3, I found that the teacher education curriculum is strongly influenced by Shulman's (1986) Theory of Pedagogical Content Knowledge and Ball et al.'s (2008) Theory of Mathematical Knowledge for Teaching. Thus, my interest in researching the understanding primary pre-service teachers develop of the knowledge necessary to carry out tasks of teaching mathematics from the perspective of Ball et al. (2008) and Ball (2017), was born. While this theory is discussed in Chapter 2, the ontology and the epistemological views of the researcher will not be further elaborated, as this is beyond the scope of this thesis. As noted, the work's focus is directed towards how pre-service teachers develop their understanding of the knowledge necessary to teach mathematics during teacher education and how that can inform the design of pre-service teachers' education in mathematics.

1.5 Structure of the Thesis

This thesis comprises the synopsis and four articles. In this first chapter, the research context and aim were delineated, followed by the research questions, and a brief discussion of the articles comprising the synopsis. The second chapter begins with a review of pertinent literature, as this provides the justification for the theoretical and methodological approaches adopted during data collection and analysis. In Chapter 3, the study is further contextualized by detailing the primary school education system and primary teacher education in Malawi. The research design is presented in Chapter 4 along with the data collection instruments and the analytical models adopted to interpret the data. In Chapters 5 and 6, the four articles comprising the synopsis are presented and discussed, focusing on common findings and key topics. Finally, the thesis closes with Chapter 7, where potential contributions of the present study to the mathematics teacher education as well as the extant research on teacher education are outlined. Study limitations are also indicated based on several suggestions for further research.

2 Theoretical Background and Relevant Research

2.1 Teaching Practice and Teaching Knowledge

Can the skills and characteristics required of an effective teacher be taught? Are some ways of preparing teachers better than others? What is involved in the practice of teaching? Although teaching in one form or another has existed throughout human history and extensive research on teaching and teacher education has been conducted over the years, these and other questions persist.

Axelrold (1973) described teaching as a didactic or evocative activity. The didactic teaching emphasizes teachers' responsibility for transmitting pertinent knowledge or instructing others on how to do something. This teaching model is typically employed by teacher craftsmen (Axelrold, 1973) who have full control of the learning environment, and are solely responsible for the students' learning and the direction that the lesson takes in the classroom. In other words, didactic teachers allow learning to occur (Novak, 1998). As a result of this hierarchical and rigid process, learners' focus is directed on memorizing facts and prescribed procedures, without seeking to understand the broader context or draw conclusions. The evocative teaching, on the other hand, emphasizes the role of teachers as teacher artists (Axelrold, 1973) whose aim is to enable learners to take control of their learning process and create evocative situations that promote learning. In this teaching model, the emphasis is on "inquiry" and "discovery" due to which lessons are designed to respond to the students' needs and aspirations with the emphasis on creativity, improvisation, and expressiveness (Gage, 1978).

Describing teaching without considering learning is to understand the work of the teacher only partially. According to Hiebert and Grouws (2007), teaching consists of "classroom interactions among teachers and students around content directed toward facilitating students' achievement of learning goals" (p. 372). This definition encompasses the ways in which multiple features that contribute to defining teachers' roles impact students' learning. Therefore, to understand the function of teachers and the effectiveness of teaching, it is necessary to understand the kind of learning goals the teaching is designed to achieve.

When bringing such a viewpoint to the context of teacher education, both teaching and learning dimensions should be considered. Pre-service teachers with or without teaching experience are simultaneously aspirant teachers and students. Although they are defined as teachers who had not yet completed a degree course in teaching, research has shown that pre-service teachers also differ from experienced teachers in terms of the beliefs they hold (Wideen, Mayer-Smith, & Moon, 1998). Pre-service teachers often begin their education with an intuitive idea about teaching that is established from their previous experience in schools (Barkatsas & Malone, 2005; Wilson, Cooney, & Stinson, 2005). During teacher education, pre-service teachers are exposed to many new ideas of teaching as they take theoretical courses and have supervised teaching practice in schools (Lavigne, 2014). After becoming in-service teachers, they are forced to modify their pedagogical beliefs because of contexts and tasks designed (Lavigne, 2014; Sheridan, 2016). The concepts of teaching and learning take on different shapes as the pre-service teachers become in-service teachers (Ng, Nicholas, & Williams, 2010).

Although teacher education has been studied extensively, the focus is typically given to the cognitive difference between what teachers should learn and what they should be able to do (Darling-Hammond & Bransford, 2005). In this context, the works of Shulman (1987), Perrenoud (1993), Freire (1996), and Tardif (2002) are particularly noteworthy. Even though these authors used similar terms to convey the same meaning, they used different approaches to study teacher preparedness. For example, Perrenoud (1993) and Freire (1996) focused on the teaching practices and roles of teachers in classrooms, whereas Shulman (1987) and Tardif (2002) direct their attention to the education and professionalization of teachers (Neto & Costa, 2016). As noted by Fernandez (2014), most of Shulman's work was dedicated to developing "the body of understanding and skills, and device and values, character and performance that together constitute the ability to teach" (p. 82). Based on his findings, Shulman (1986) opined:

> The teacher needs not only understand that something is so; the teacher must further understand *why* it is so, on what grounds its warrant can be asserted, and under what circumstances our belief in its justification can be weakened and even denied. Moreover, we expect the teacher to understand why a given topic is particularly central to a discipline whereas another may be somewhat peripheral. This will be important in subsequent pedagogical judgments regarding relative curricular emphasis. (p. 9)

According to this perspective, the work of teaching entails tasks that teachers must execute to help students to learn (Shulman, 1986). Teachers must be able to determine the content that is essential to meet students' learning needs and specificities. This means that once the content's essence, origins, and the logic-historical processes that justify the existence of the content are understood, teachers should be able to orient their students' learning beyond simple facts and predetermined standards, which is a prime condition for effective teaching (Grossman et al., 2009).

To meet the goals outlined above, Shulman (1986) suggested that teachers needed to possess three categories of knowledge to teach a particular subject effectively: subject matter knowledge, curricular knowledge, and pedagogical content knowledge. The first category—subject matter knowledge—refers to "the amount and organization of knowledge per se in the mind of the teacher" (Shulman, 1986, p. 9). According to Shulman (1986), an effective teacher should know not only the facts and concepts pertinent to the domain, but should also be able to explicate why the domain is worth knowing and how it relates to other domains. The second category—curricular knowledge—entails awareness of what the curriculum proposes and the norms and principles of the work setting. In other words, curricular knowledge involves knowledge about the programs of study and curricular materials used to teach a subject, as this allows teachers to make connections between previously studied material and topics to be introduced later in the learning process, which is an essential aspect of teaching (Brant, 2006).

The third category—pedagogical content knowledge—refers to the knowledge base of teaching at the intersection between content and pedagogy (Shulman, 1986). Such knowledge, according to Shulman (1986), encompasses “aspects of content most germane to its teachability” (p. 9). It includes the ability to identify and organize concepts presented in class (representations, analogies, illustrations, examples, explanations, and demonstrations) to make a subject more comprehensible for the students.

Even though Shulman (1986) proposed the aforementioned ideas about pedagogical content knowledge nearly 35 years ago, these conceptualizations have gained momentum in recent investigations about teacher knowledge. His work has also served as a basis for the recent educational reforms and has influenced research efforts and educational policies in several countries. In recent years, pedagogical content knowledge is increasingly being taught by teacher educators in teacher educational programs, especially those aimed at primary school education. Given that Shulman (1986) conceived pedagogical content knowledge in general terms, his ideas have since been expanded to help teachers learn and develop a better sense of the tasks and knowledge demanded for teaching subject matter.

As a part of this research initiative, Shulman’s (1986) ideas have been investigated in the context of pre-service mathematics teacher education. An ample body of frameworks has been produced on this topic, including the works of Ball et al. (2008), Chevallard’s (2000), Davis and Simmt (2006), and Rowland, Huckstep, and Thwaites (2005). Although aligning with Shulman's (1986) ideas, these frameworks have pursued different ideas and approaches regarding teaching knowledge, including examining associations between mathematical knowledge and practice (Chevallard, 2000), investigating the complex dynamics of the mathematical knowledge that teachers needed for teaching (Davis & Simmt, 2006), studying the differences between content knowledge and pedagogical content knowledge and implications for teaching and learning (Baumert et al., 2010; Krauss et al., 2008), and exploring different aspects of teacher knowledge that contribute to the professional development of pre-service teachers (Rowland et al., 2005).

The framework of Ball et al. (2008), in particular, focuses on representations of the knowledge entailed in the work of mathematics teachers. Such a framework of mathematical knowledge for teaching comprises the areas that are unique to the role of mathematics teacher by examining how subject matter and pedagogical content knowledge are employed to carry out the tasks of teaching mathematics (Ball et al., 2008).

Additionally, Ball et al.'s (2008) works focus on the recurrent tasks and problems of teaching mathematics, what teachers do as they teach mathematics, and the mathematical knowledge, skills, and sensibilities required to manage these tasks. A list of the tasks identified as the tasks entailed in the work teachers do when they are teaching mathematics includes:

- Presenting mathematical ideas,
- Responding to students' "why" questions,
- Finding an example to make a specific mathematical point,
- Recognizing what is involved in using a particular representation,
- Linking representations to underlying ideas and other representations,
- Connecting a topic being taught to topics from prior or future years,

- Explaining mathematical goals and purposes to parents,
- Appraising and adapting the mathematical content of textbooks,
- Modifying tasks to be either easier or harder,
- Explaining the plausibility of students' claims (often quickly),
- Giving or evaluating mathematical explanations,
- Choosing and developing usable definitions,
- Using mathematical notation and language and critiquing its use,
- Asking productive mathematical questions,
- Selecting representations for particular purposes, and
- Inspecting equivalencies.

The tasks outlined by the authors are examples of what is required for teachers to carry out to conduct their teaching successfully. They reveal the complexity and dynamic of activities that regularly occur in the classroom and offer a window into the knowledge entailed in teaching mathematics in broader contexts (Ng, Mosvold, & Fauskanger, 2012; Selling, Garcia, & Ball, 2016).

In analyzing these tasks, Ball et al. (2008) were guided by the empirical evidence supporting the existence of six domains of teaching knowledge needed to carry out the tasks of teaching mathematics effectively. These domains were typically denoted as common content knowledge (CCK), specialized content knowledge (SCK), horizon content knowledge (HCK), knowledge of content and students (KCS), knowledge of content and curriculum (KCC), and knowledge of content and teaching (KCT), and their organization into systematic units as presented in Figure 1.

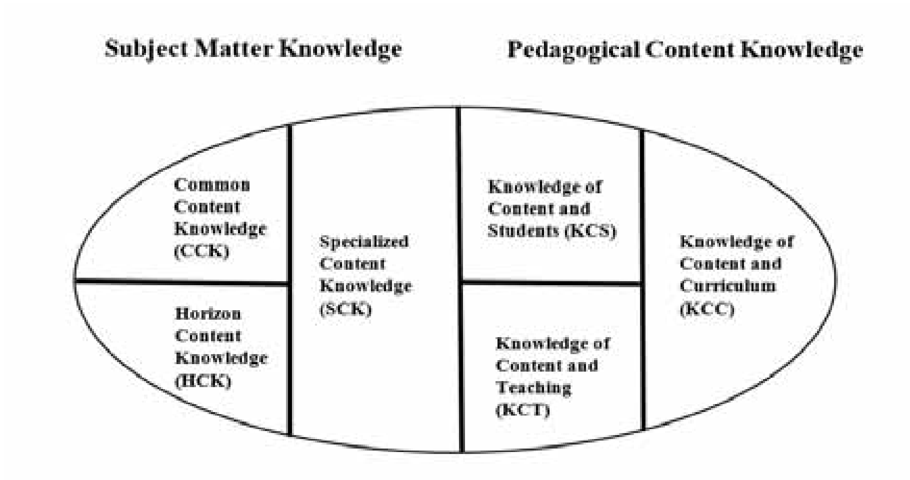


Figure 1. Framework of mathematical knowledge for teaching (Ball et al., 2008, p. 403).

According to Ball et al. (2008), CCK domain refers to the knowledge that is common in a wide variety of settings, rather than pertaining solely to the work of teaching. For example, engineers or economists use this type of knowledge to solve problems in their daily work. Similarly, using an algorithm to find the answer for a subtraction problem is an example of CCK. In teaching, CCK allows teachers to appropriately respond to students' questions and resolve any misunderstandings related to the subject matter (Ndlovu, Amin, & Samuel, 2017).

SCK, one the other hand, is the knowledge unique to the work of mathematics teaching. It "involves an uncanny kind of unpacking of mathematics that is not needed—or even desirable—in settings other than teaching" (Ball et al., 2008, p. 400). Some examples of SCK include the knowledge needed to carry out tasks of teaching unique to the work of teaching such as introduce mathematical concepts in a way that is accessible to the students (Ball et al., 2008). For instance, when introducing students to the notion of numbers, the teacher needs to know how the students perceive this concept in various real-world contexts. As noted by Worden (2015), this necessitates not only the capacity for "transforming content knowledge into pedagogical content knowledge but also unpacking one's content knowledge to make it available for such transformation" (p. 106).

The CCK and SCK domains are interrelated via the HCK domain, which is defined as the mathematical knowledge from a broad perspective (Ball et al., 2008). Thus, HCK entails knowledge of the discipline, its origins, and the value of curriculum in its multiple dimensions and settings (Jakobsen, Thames, Ribeiro, & Delaney, 2012). As this necessitates the general knowledge of the previous and forthcoming content, it is often equated with "a peripheral mathematical vision needed in teaching" (Hill, Rowan, & Ball, 2005, p. 70). In teaching practice, HCK allows teachers to develop a sense of conceptual nexus between the curriculum and a broader perspective of the discipline (Jakobsen et al., 2012).

The KCC domain combines the knowledge of mathematics and the curriculum, as conceived by Shulman (Sleep, 2009). This domain also includes the skills required to effectively use the teaching materials such as textbooks and didactic materials, teaching instruments such as Blackboard, and technology such as calculators and computers (Koponen, Asikainen, Viholainen, & Hirvonen, 2016).

The KCS domain represents an amalgam of knowledge of content and students (Ball et al., 2008). It implies the capacity for anticipating how students will interpret the taught material and which aspects they will find difficult to understand. To meet these aims, teachers must be able to hear and respond to students' arguments and choose instruction approaches that promote student learning. Consequently, KCS also necessitates the awareness of students' motivation and aptitude for learning mathematical topics.

Finally, KCT combines knowledge of mathematics and teaching, in recognition of the fact that, in order to teach mathematics effectively, teachers must be able design lessons appropriately. This includes proper selection of activities, exercises, and representations for different topics. One crucial characteristic of this knowledge is the teacher's ability "to recognize situations where teachers should diverge from their original planning, for example, if a student makes a mathematical discovery" (Koponen et al., 2016, p. 152).

The six domains presented above imply that the integration of knowledge types is unique to mathematics teachers (Ball & Bass, 2000). Teaching mathematics includes a core of tasks that teachers must carry out to help students to learn (Ball & Forzani, 2009). Such tasks are complex and reveal qualities that other professions do not demand. The work of mathematics teachers is a specific activity that differs from casual actions including commonplace showing, telling, or helping (Cohen, 2011; as cited in Ball & Forzani, 2009). For example, although an engineer possesses high-level mathematics knowledge and at least reasonable science knowledge, the engineer can only provide information or show one another how to do things. The mathematics teacher, on the other hand, aims at the professional classroom teaching (Ball & Forzani, 2009), an endeavor that includes the creation of opportunities for students to learn and develop their understanding of the subject matter. In this sense, the teacher's role is driven by social and moral conduct and a human sense to help students develop their best qualities as human beings (Jacinto & Cedro, 2012).

Teaching mathematics requires specialized knowledge and skills that go beyond subject matter alone, and Ball et al.'s (2008) Theory of Mathematical Knowledge for Teaching provides the analytical tools to identify and analyze the kind of knowledge and skills that mathematics teaching actually requires (Ding, 2016; Goos, 2013; Jakobsen et al., 2012; Stephenson, 2018). However, it also has attracted considerable criticism due to its limited application on how the framework could be useful for guiding teachers to teach mathematics (Mitchell, Charalambous, & Hill, 2014) or provide better insights into teachers' views and understandings of the mathematical knowledge for teaching (Mosvold & Fauskanger, 2013). This is a particular challenge for the field of teacher education and teaching knowledge since the quality of teaching and teaching knowledge depends on the views and understandings of those who actually teach. Therefore, this thesis seeks to provide insights into the understanding that pre-service teachers develop of the knowledge necessary to teach mathematics.

The following sections aim to contextualize the current study into the research field of teaching knowledge. They focus on relevant research related to teachers' beliefs of teaching knowledge, followed by a clarification of the main terms used in this study.

2.2 Beliefs of Teaching Knowledge

Over the last few decades, a growing body of research has been conducted to elucidate teachers' mathematical knowledge needed for teaching (Hill et al., 2008). However, most of the extant studies have focused on the ways teachers' knowledge and beliefs influence student performance (Hill et al., 2005; Rockoff, Jacob, Kane, & Steiger, 2011) and instructional practice (Ball, 1990; Ben-Peretz, 2011; Fennema & Franke, 1992; Mapolelo & Akinsola, 2015; Wilkins, 2008). In particular, only a small portion of these studies have focused on teachers' views and understanding of the knowledge needed for teaching mathematics (Hatisaru, 2018; Mosvold & Fauskanger, 2013).

Research on teachers' beliefs about teaching knowledge, while limited, has yielded some valuable findings (Ferguson & Brownlee, 2018; Fives & Buehl, 2008; Hofer, 2002; Mosvold & Fauskanger, 2013; Sinatra & Kardash, 2004). Fives and Buehl (2008), for instance, described the concept of personal epistemology in the context of studies about teaching knowledge. The authors employed qualitative and quantitative methods to examine pre-service teachers' and practicing teachers' beliefs about teaching knowledge and teaching ability. While the authors provided valuable insights for developing a framework to conceptualize teachers' beliefs about teacher knowledge, they called for further investigations using longitudinal and cross-sectional methodologies to explore this topic further, as "such studies would indicate whether these beliefs are developmental in nature and change as one experiences the profession" (Fives & Buehl, 2008, p. 172).

Drawing upon the work of Fives and Buehl (2008) and Philipp's (2007) concept of belief, Mosvold and Fauskanger (2013) explored the epistemic beliefs that teachers have about the knowledge needed to teach mathematical definitions. The researchers gathered pertinent data via focus-group interviews involving 15 pre-service and in-service teachers in Norway, which was subjected to content and inductive analysis. Their findings revealed that, while some teachers believed that knowledge of definitions is an integral part of their mathematical knowledge for teaching, others opined that the mathematical definitions are important for higher grades but are not necessary for lower-grade students. The participating teachers were, however, aware of the cultural differences in accepted mathematical definitions.

In a subsequent study, Mosvold and Fauskanger (2014) focused specifically on the domain of mathematical horizon content knowledge. In this context, the authors discussed the beliefs pre-service and practicing teachers have about the knowledge at the mathematical horizon for teaching. A significant finding that emerged from this study was that teachers did not seem to emphasize HCK in their education and practice. When discussing aspects of broader content, participants tended to focus mainly on whether a particular mathematical content was directly related to the curriculum for a specific grade level. This investigation illustrates difficulties encountered when investigating teachers' beliefs about mathematical knowledge for teaching, as this is a complex phenomenon that some teachers might find difficult to articulate.

In other studies, focus was primarily given to specific characteristics of teaching knowledge. For instance, Leikin and Zazkis (2010) interviewed secondary school teachers about their usage of advanced mathematical knowledge acquired during undergraduate studies at colleges or universities. The authors adopted a qualitative approach based on grounded theory (Strauss & Corbin, 1990), aiming to identify common themes in the teachers' data. Their findings indicate that most teachers acknowledge the relevance of advanced mathematical knowledge but have difficulties in generating specific problems or recalling situations in which advanced mathematics knowledge can be useful (Leikin & Zazkis, 2010). In particular, only a few participants were able to provide content-specific examples for the purposes and advantages of their advanced mathematical knowledge for student learning, such as personal confidence, and the ability to make connections and respond to students' questions (Leikin & Zazkis, 2010). Based on these findings, the authors called for a more articulate relationship between advanced mathematical knowledge and mathematical knowledge for teaching.

While the research briefly reviewed in the preceding sections is relevant to the understanding of how teaching knowledge influences the quality of teaching, how teaching knowledge functions in the teaching-learning process of pre-service teachers during teacher training remains to be established. Extant studies on this topic suggest that teaching knowledge should be examined through the lens of pre-service teachers' perceptions of knowledge domains (Kilic, 2015), their self-perceptions of the tasks of teaching (O'Meara, Prendergast, Cantley, Harbison, & O'Hara, 2019), and their views on and understanding of the certainty of teaching knowledge (Ferguson & Brownlee, 2018). Empirical evidence shows that teachers' beliefs have a strong influence on the way they approach students' specificities and learning needs (Givvin, Stipek, Salmon, & MacGyvers, 2001), comprehend mathematical knowledge (Cady & Rearden, 2007), and develop their identity as a teachers (Ponte, 2011). Thus, there is a need to examine whether pre-service teachers truly understand the teaching tasks and the knowledge needed to carry out these tasks during mathematics instruction.

Influenced by the works of Mosvold and Fauskanger (2013) and Fives, Lacatena, and Gerard (2015), the present study focuses on the pre-service teachers' understanding of the knowledge defined as relevant to the practice of mathematics teaching. Thus, its aim is to elucidate how pre-service teachers develop their understanding of the knowledge necessary to teach mathematics throughout teacher education.

Having presented the theoretical background of this study, it is now important to clarify the concept of belief and the reasons for adopting *understanding* as the focus of this study.

2.3 Clarification of the Terms: Belief and Understanding

Pre-service teacher's beliefs are a critical part of professional preparation for the work of teaching, as discussed previously in this thesis. It is particularly relevant to researchers aiming to elucidate the central aspects of teachers' roles and attitudes (Akinsola & Awofaba, 2009; Minor, Onwuegbuzie, Witcher, & James, 2002), instructional practices (Levitt, 2001; Thompson, 1992), and the conceptions and views related to teaching and learning specific subjects (Ball, 1990; Beswick, 2012). The purpose of the forthcoming discussion, however, is not to diminish the value of studies exploring teachers' or pre-service teachers' beliefs about teaching, but rather to highlight the emergent need to investigate their understanding of crucial characteristics of the knowledge and skills entailed in the work of mathematics teaching.

The notion of belief has received multiple connotations over time (Pajares, 1992). Looking at its linguistic roots, the word belief comes from Old English lēafa "faith"—a later variant to Proto-Germanic ga-laubon that means "told dear, esteem, trust" (Harper, 2016). During the 12th century, the word "belief" was used synonymously with religious-like faith (Stringer, 1996). The evolution of the use of the word belief in the English language has been thoroughly investigated with respect to Christianism's influence and the dynamics of power that have influenced its meaning. Based on the findings yielded, belief is widely accepted as a Christian concept, due to which, its use in other contexts may raise translational and cultural concerns. A non-English speaking person, for instance, from a culture without a notion of belief, might interpret it as a vague proposition (Needham, 1972), a social style (Lindquist & Coleman, 2008), or an experiential efficacy of practice (Mazzoni & Kirsch, 2002). These multiple overlapping meanings can lead to serious misinterpretations and misuse of the concept of beliefs depending on the context.

In the educational research literature, the concept of belief has been defined and operationalized in a variety of ways, giving rise to concepts such as epistemic belief (Bråten, 2010), epistemological belief (Schommer, 1990), personal epistemology (Sandoval, 2005), and epistemic cognition (Moshman, 2016). Epistemic beliefs can be defined as a lens through which a person interprets a phenomenon or situation (Rebmann et al., 2015) or as "psychologically held understandings, premises, or propositions about the world that are thought to be true" (Philipp, 2007, p. 259). It also refers to the knowledge about how people justify and associate matters of objectivity, subjectivity, rationality, and truth (Moshman, 2016). Even though a variety of definitions are adopted in extant studies, in general, researchers equate epistemic beliefs with what individuals know about the nature of knowledge and the process of its acquisition (Conley, Pintrich, Vekiri, & Harrison, 2004; Fives & Buehl, 2008;; Hofer & Pintrich, 1997; Schommer, 1994).

In the educational research field, teachers' epistemic beliefs are usually examined in relation to the practical aspects of mathematics instruction (Bendixen & Feucht, 2010). This research stream has given rise to numerous thought-provoking questions such as: what do teachers think they need to know about teaching? How can teachers' beliefs about the nature of knowledge be utilized to understand and improve educational strategies? and how do teachers' beliefs about knowledge and knowing develop over time? In addition to addressing these questions, most authors recognize the significant need for teachers to reflect on and be aware of their practice (Fives & Buehl, 2008).

The present thesis contributes to the extant body of research on teachers' epistemic beliefs about the knowledge for teaching mathematics. The main focus is on a single component of Philipps' (2007) concept of epistemic belief, which includes an understanding of the nature of knowledge and knowing. The main research question addressed in the present study is: How do pre-service teachers develop their understanding of the knowledge necessary to teach mathematics throughout teacher education? In answering this question, understanding is treated as an epistemic dimension of the belief concept.

Understanding is a crucial condition for the development of epistemic beliefs. Yet, defining understanding is a complex task, as it might be perceived as a species of knowledge (Grimm, 2006). In pertinent literature, different forms of understanding have also been considered, most of which involve a cognitive achievement (De Regt, 2004; Kvanvig, 2003), a mental process or a state of consciousness (Wittgenstein, 2007), or epistemic goods (Haddock, Millar, & Pritchard, 2009). In the present study, however, Smith and Siegel's (2004) views of the concept of understanding are adopted as the starting point. These authors defined understanding under four conditions, each of which is deemed necessary for understanding.

The first criterion of understanding is *connectedness*, which is defined by Smith and Siegel (2004) as the process by which concepts and ideas are linked together. According to this view, the ability to connect different ideas, subjects, or phenomena is an indication of a person's understanding. For instance, understanding the concept of real numbers requires that students identify and define the concepts of natural, whole, integers, and rational and irrational numbers as subsets of the real number set, as well as provide appropriate explanations for the connections among these concepts. In addition, they should be able to make connections between abstract concepts and practical experience. As pointed out by Bassarear (1997), one "can truly understand an idea only if it is well-connected to other ideas" (p. 9). Based on this view, one's depth of understanding depends on how many connections exist and the quality of those connections (Bassarear, 1997).

The second criterion is the *attribution of meaning* or in Smith and Siegel's (2004) words, "sense-making" (p. 563). This condition relates to the ability to generalize a concept or idea into different settings. It can thus be interpreted as a subjective relationship that an individual has with a concept, object, or phenomenon. For example, a person might know what real numbers mean, but (s)he might find no reason to use such knowledge in everyday life. Depending on the context or circumstances, if a real number has no value for an individual, he or she will likely make no sense of it.

These assertions lead to the third criterion denoted as *application*. According to Smith and Siegel (2004), this condition refers to the ability to apply a concept or idea in different situations (such as problem-solving situations). In the process of learning mathematics, for instance, this condition involves relying on the existing knowledge and skills to solve a specific problem. Using the earlier example, if a person knows what real numbers are but is unable to use such knowledge appropriately in a particular situation, such understanding is incomplete. It is also noteworthy that the condition of application consists not only of the ability to provide explanations of what real numbers mean but also how they can be appropriately employed to the encountered contexts and problems.

The fourth criterion, *justification*, involves a coherent appraisal of at least some of the reasons to support a claim. Smith and Siegel (2004) opined that one must be able to evaluate a concept, idea, or situation that is present. For example, students might know the basic facts about real numbers and might be able to respond to questions about them without actually truly understanding the concept of real numbers. However, in some situations, they will likely not be able to justify the correctness of their solutions or provide an example that helps to make sense of them since reasoning demands more than merely knowing mathematical facts and procedures.

The four conditions of understanding proposed by Smith and Siegel (2004) highlight not only the major goals of science education but also acknowledge that science (mathematics in particular) teaching should aim to develop one's knowledge and understanding of the concepts and content without the endeavor in having to change the students' beliefs. This strategy recognizes the immense value of education and teachers in the classroom and opens a new window for education research to explore new boundaries. As findings yielded by research on epistemological beliefs have made a significant contribution to education, the work presented in this thesis is guided by the premise that the concept of understanding—defined here as an epistemic component of the belief concept—can provide an alternative interpretive lens (or conceptual instrument) for examining teachers' and pre-service teachers' learning process, particularly, the pre-service teachers' ways of interpreting, constructing, and using the acquired knowledge to conduct the tasks of teaching properly.

3 Context of the Study

This chapter introduces the context in which this study took place. Most of its content is based on a limited version of a book chapter (Jacinto, Kazima, Jakobsen, & Bjuland, in press, p. x) written to complement the current research. This book chapter describes the reality of teacher education in Malawi and provides a historical background of the actions taken to qualify pre-service teachers for teaching in Malawi. It also gives an overview of the current programs, institutional documents, and approaches for preparing pre-service teachers to teach mathematics in primary schools. Such a content is now part of this thesis and contributes to our understanding of the processes via which pre-service teachers must pass to become teachers in Malawi.

Teacher education has been an important political instrument for improving the quality of the educational system in societies, as pointed out in Chapter 1. Despite progress, many of its nations are struggling to meet the targets set by the United Nations (UN) on Sustainable Development Goal 4 (SDG4) to ensure inclusive and equitable quality education and promote lifelong learning opportunities for all (UNESCO, 2016a). In the region of Sub-Saharan Africa, nearly 7 out of 10 countries face shortages of teachers (about 6 million teachers) to achieve universal primary and secondary education (UNESCO, 2015a). By 2030, 2.2 million new teaching positions need to be created to meet the targets, while filling about 3.9 million positions due to attrition (UNESCO, 2015b).

In the Republic of Malawi, the first Sub-Saharan African country to implement free primary education, the quality of education has been compromised by the employment of teachers with little or no training (United Nations International Children's Emergency Fund [UNICEF], 2019). Most of the Sub-Saharan African countries are members of the Southern Africa Consortium for Monitoring Educational Quality

(SACMEQ), which is a consortium of Ministries of Education in conjunction with UNESCO's International Institute for Educational Planning. The consortium aims to work together to share experiences and apply scientific methods for monitoring and evaluating the quality of education in the participating countries, specifically Botswana, Kenya, Lesotho, Malawi, Mauritius, Mozambique, Namibia, Seychelles, South Africa, Swaziland, Tanzania (Mainland), Tanzania (Zanzibar), Uganda, Zambia, and Zimbabwe. SACMEQ administered tests in numeracy and literacy to primary grade 6 students in all the participating countries. The first test was in 1995, the second in 2000, and the third in 2007. According to the third SACMEQ test, seven in ten children in Malawi lack proficiency in reading and mathematics (SACMEQ, 2010). This situation puts Malawi as one of the least performing countries in Sub-Saharan Africa (Ministry of Education, Science, and Technology, 2016). The details of performance by Malawian grade 6 students found that more than 90% of the students were operating at a basic numeracy level, which is three grade levels below the expected level of achievement (SACMEQ, 2010).

Kazima, Jakobsen, and Kasoka (2016) describe many factors that contribute to Malawian students’ low achievement, including large class sizes, limited teaching and learning resources, and the teacher’s poor quality. Although these factors are connected and need to be addressed to improve the quality of education in Malawi, the quality of teachers is most important because a well-qualified teacher will be able to cope and teach better within the limited circumstances of the Malawian context than an unqualified teacher (Kazima, 2014; Kayuni & Tambulasi, 2007). Furthermore, other factors also affect countries like Lesotho and Zimbabwe, whose students performed better than students in Malawi. Hence, the quality of teaching seems to be the main factor for Malawian students’ low achievement. This is likely to be a consequence of employing unqualified teachers to meet the high demand for teachers that was created by the introduction of free primary education.

3.1 Background: The Republic of Malawi

Malawi is a landlocked southern African country that became a republic in 1966 following its independence from Britain in 1964. Prior to independence, Malawi was called Nyasaland and was a British protectorate (Chanaiwa, 1993). From independence in 1964 to 1994, there was only one party, and the leadership was not democratic (Kazima, 2014). In 1994, other parties were introduced, the first presidential and multiparty elections were held, and a new party came into power. Since then, Malawi has been a multiparty democratic nation, and so far, three different parties have been in power. Malawi is densely populated. It has an area of approximately 118,500 square kilometers and a population of about 18.7 million people (Government of Malawi, 2018). Nearly one-tenth of the population lives in urban areas (11.4%), whereas the remaining majority live in rural areas (88.6%; Yaya, Bishwajit, & Shah, 2016). Malawi is also one of the world's poorest countries, ranked as 171 out of 189 countries on the Human Development Index (United Nations Development Program, 2018).

Figure 2. Malawi and the Sub-Saharan Africa region.

Malawi has struggled to provide quality education in schools due to the large population and the limited resources available. Consequently, in past years, Malawi has become a focus of concern for world leaders due to the poor indices in reading and mathematics competencies in schools (SAQMEQ, 2010). With the introduction of free primary education in 1994, Malawi now faces multidimensional challenges such as lack of facilities, high pupil/teacher and pupil/classroom ratios, low learning achievement, and poor qualification of teachers. In urban districts, the average number of pupils per teacher is about 60. In contrast, in rural areas of the country, such as the Mangochi district, the rate reaches 1 teacher per 152 pupils (UNICEF, 2019). Over the years, the government has introduced some interventions in an effort to address the problems. These have included double shifts in schools to address the problem of classroom space and hiring many unqualified teachers to meet the high demand for teachers (UNICEF, 2019).

3.2 The Malawian Education System

The Malawian education system is divided into three levels: primary, secondary, and tertiary (postsecondary) education. In primary school education, children's ages range from 6 to 14 years old, ascending from Standard 1 to Standard 8. Secondary school education is offered for students with ages between 15 to 19 years old from Form 1 to Form 4. However, due to students repeating a grade and other reasons, the age range might exceed 14 and 19 for primary and secondary school, respectively. The duration of tertiary education varies depending on the programs and institutions. For example, programs at a university often take four to five years, and vocational education can take one to two years. Pre-primary education in Malawi is a non-mandatory program designed for children ages 3 to 5 and offered by Early Childhood Development Centres that are managed by the Ministry of Gender, Women, and Child Welfare.

3.3 Primary Teacher Education in Malawi

Primary teacher education in Malawi started during the colonial period. Primary teacher colleges were established by churches in association with the Christian Council (Banda, 1982), and all the teacher colleges followed a three-year program with the same curriculum. The three-year curriculum contained more content courses than the current curriculum. After independence, the Ministry of Education took over responsibility and management of primary teacher education and established more teacher-training colleges (TTC). As of 2019, eight teacher-training colleges exist.

Since the introduction of free primary education in 1994, there has been a rapid increase in the number of children in primary schools from 1.8 million in 1993 to 4.5 million in 2014 (Government of Malawi, 2016). This resulted in a very high demand for teachers and the Ministry employed approximately 22,000 unqualified teachers with a plan to offer on-the-job training. The minimum requirement for the recruitment was a Junior Certificate which was obtained after passing national examinations at the end of two years of secondary education. Therefore, these teachers had only two years of secondary school mathematics which include Algebra, Euclidean Geometry, and Arithmetic. Consequently, the teachers' own knowledge of mathematics was not much higher than the primary school mathematics they were required to teach.

As another intervention to address the shortage of qualified teachers, the Ministry reduced the teacher education program's duration from three years to two years (Kazima, 2014). After recruiting thousands of unqualified teachers, the Ministry discontinued the fulltime program for teacher education and introduced a largely school-based program in 1997, called the Malawi Integrated In-Service Teacher Education Program (MIITEP). The program took twenty-four months with four months for college-based courses and the remaining 20 months for teaching in schools. The college-based courses included Educational Foundations, English, Mathematics, Science, and Expressive Arts. The mathematics included mostly Arithmetic and mirrored the topics of primary school mathematics. The school-based learning was in the form of teaching practice with supervision by the college lecturers and the classroom teachers. The college lecturers were recruited by the Ministry of Education and were required to have more than 10 years of teaching experience in primary schools and a minimum qualification of a Diploma in Education, which is lower than Bachelor's degree level.

The MIITEP was designed to be an emergency program for about five years. However, it continued to run for ten years until 2005 since more unqualified teachers continued to gain employment. In 2005, the Ministry discontinued the MIITEP and introduced a new program called the Initial Primary Teacher Education Program (Ministry of Education, 2005).

The new Initial Primary Teacher Education Program (IPTE) had a duration of two years. The first year consisted of full-time college-based courses, and the second year was full-time school based in the form of teaching practice. The entry requirement into the program was also revised from the Junior Certificate to the Malawi Schools Certificate, which requires a passing national examination after completing the four years of secondary education. The examinations are equivalent to O-Level in the English system. The candidates for this certificate would have covered Algebra, including algebraic expressions, linear equations, quadratic equations, graphs of equations, and inequalities; Euclidean Geometry including lines and angles, polygons and their properties, circle properties and theorems; Arithmetic, specifically commercial arithmetic; and Trigonometry, including sine, cosine, tangent, and trigonometric identities. The curriculum of the IPTE Program had a total of ten learning areas: agriculture, education foundation studies, environmental studies, expressive arts, general and social studies, life skills, literacy and languages (English and Chichewa), numeracy and mathematics, religious studies, and science and technology. All pre-service teachers were required to study all ten learning areas since the primary school teachers are expected to teach all the subjects in primary school (Ministry of Education, 2005). This curriculum was offered from 2005 to 2016.

In 2017, the IPTE curriculum was revised, and the structure of the program changed from one year (three terms) of college courses and three terms of teaching practice to four terms of college courses and two terms of teaching practice. In this new structure, pre-service teachers take theoretical courses in college for the first two out of three terms of the first year, learning subject content with a special focus on teaching methods for primary classes. After completing the coursework in these two terms, pre-service teachers go into the field for practice, experiencing teaching in both lower and upper primary classes for two terms (last term in the first year, and first term in year two). During teaching practice, the students are supervised periodically by the teacher educators and more regularly by mentors in the school. Mentors are experienced teachers who have been trained by the teacher colleges to supervise student teachers during teaching practice in the schools. In the last two terms of year two, the pre-service teachers return to college to continue learning subject content to reflect on their experience of teaching practice then complete their studies (Malawian Institute of Education, 2017).

The current revised curriculum (Malawian Institute of Education, 2017) has adopted a reflective practitioner model of teacher education (Schön, 1987) that aims to connect practice and theory and integrates content and pedagogy in teaching and learning. The innovative features of this curriculum include:

- The curriculum design is based on reflective practice principles;
- Introduction of specific early grade teaching methodologies;
- Delivery of the subject content following modular approaches;
- Teaching experience in both lower classes and upper classes of primary school; and
- Cross-cutting studies involving assessment for learning, information and communication technology, and critical thinking.

The primary teacher education curriculum is designed in a modular structure and contains the same ten learning areas outlined earlier in this chapter, but with different weighting in terms of time. Three learning areas (Education Foundations Studies, Numeracy and Mathematics, Literacy and Languages) are allocated more time than the others. In this modular design, a set of topics forms a module in a subject. A module consists of 40 hours of contact time. The core elements in Numeracy and Mathematics are shown in Table 2.

Table 2. Core elements in numeracy and mathematics by term and number of hours (Malawian Institute of Education, 2017).

Core Element	**Term 1**	**Term 2**	**Terms 3 and 4**	**Term 5**	**Term 6**
Theories, Concepts, and Issues in the Teaching and Learning of Mathematics	40		Teaching Practice		
Number Concepts and Operations (e.g., whole numbers, fractions, decimals, percentages, rate, ratio, proportion)	40	40			
Measurement (e.g., length, perimeter, area, volume)		40			
Data Handling (e.g., representing data, bar graphs, pictographs)				40	
Space and Shapes (e.g., triangles, rectangles, circles)				40	
Accounting and Business Studies (e.g., profit and loss, simple and compound interest)					40
Patterns, Functions, and Algebra (e.g., number patterns, linear equations)					40

Primary teacher education programs have both continuous (formative) and summative assessments to measure pre-service teachers' achievement of knowledge, skills, values, and attitudes. These include oral presentations, practical tasks, reports, research, tests, and examinations. The grading system is given in percentages: 60% for continuous assessment and 40% for summative assessment.

The current mathematics curriculum for primary teacher education encompasses standards that require pre-service teachers to acquire subject matter knowledge and develop pedagogical ways to promote students' lifelong learning (Malawian Institute of Education, 2017). The curriculum contains three major goals for teachers in the teacher education program:

1. Develop academically well-grounded and professionally competent teachers;

2. Stimulate flexibility and capability of adapting to the changing needs and environment of Malawi society;

3. Professionalism in adhering to and maintaining the teaching profession's ethics and being imaginative in adapting, creating, and utilizing locally available resources suitable for the needs of their learners.

The IPTE curriculum rests its philosophy on the basis of the following principle: "To produce a reflective, autonomous, lifelong learning teacher, able to display moral values and embrace learners' diversity" (Malawian Institute of Education, 2017, p. ix). Therefore, preparing teachers of quality seems to be an important goal within this context, yet it is unclear how these principles and the main principles guide teachers' practice. Similarly, the syllabus for preparing primary teachers in Malawi focuses on three main skills needed for an effective teacher: competency, flexibility, and resourcefulness, but no transparency on the meaning of these skills is provided; for instance, what is meant by being "a resourceful teacher" in Malawi (Malawian Institute of Education, 2017, p. ix).

In general, primary teacher education has been an essential instrument in building a democratic system in Malawi (Kunje, Lewin, & Stuart, 2003). The existing teacher training guidelines define, to some extent, the characteristics needed by a teacher to attend to the demands of society. Recent reforms in the curriculum for preparing both primary and secondary pre-service teachers in mathematics have aimed for inclusive and equitable quality education and lifelong learning. These qualities encompass characteristics that are both unique and globally shared (profound knowledge of the subject matter and pedagogical content, abilities to work with larger classrooms and poor environmental conditions, and cultivation of specialized knowledge of resourcefulness). This information contributes to a better understanding of how the educational system for preparing elementary teachers in Malawi is constituted, which is significant to describe the context in which the current research took place.

4 Methodology

4.1 Research Design

The method chosen for this research was based on a *qualitative case study approach* (Stake, 1995). Case study is often used in educational research to provide richness and depth to the understanding of individuals, groups, and institutions, as well as their beliefs and perceptions, interactions, challenges, and issues (Hamilton & Corbett-Whittier, 2013). Although case study design can also be used in quantitative and mixed-methods research to obtain numerical information or patterns related to the phenomena observed (Creswell & Plano Clarke, 2018), it is better suited for studying the subjectivity of the investigated phenomenon (Silverman, 2005). The case study approach is, therefore, more qualitative than quantitative because it involves *causality* (George & Bennett, 2005; Starman, 2013).

The aim of a case study in qualitative research is to examine the phenomenon's particularities in relation to its totality. In educational science, a case study design is typically adopted when the aim is to answer how and why questions (Stake, 2008). Such an approach is thus beneficial for investigations that aim to go beyond descriptions and obvious inferences about a case. Since a case can be an individual, group, situation, process, or relation, it allows researchers to decode the particularities and holistic aspects of the case through the actors' perspective (Tellis, 1997). Given these characteristics, a case study approach best suits investigations that examine subjective phenomenon—such as the present investigation that examines the understanding pre-service teachers develop of the knowledge necessary to teach mathematics—that cannot be perceived through one lens, but rather from a variety of lenses (Baxter & Jack, 2008).

A case study also allows researchers to unravel "the particularities and complexity of a single case, coming to understand its activity within valuable circumstances” (Stake, 1995, p. xi). In this context, Coimbra and Martins (2013) asserted that a case study “is a social study of a phenomenon, within the unity and totality of a system, which is temporally and spatially limited” (p. 391). Such conditions allow researchers to examine not only the main characteristics of the case within its real context, but also the processes connections and dynamics that surround the case, “especially when the boundaries between a phenomenon and context are not clear and the researcher has little control over the phenomenon and context” (Yin, 2002, p. 13). Thus, the case study approach is suitable for examining the development of pre-service teachers’ understanding of the knowledge needed to teach mathematics in the context of Malawian primary education.

Studying a case in its real context allows the results to be generalized and applied to similar settings. According to Gerring (2004), a case study is best defined as an intensive study of a single unit that is generalized across a broader set of units. Similarly, Johnson (2011) stated that although the primary goal of a case study is understanding the case itself, it can also be seen as a tool to examine a “specific understanding of a smaller group of pre-service teachers in a larger number of the other pre-service teachers in the same cohort” (p. 5). Further, Johnson (2011) complemented this view by asserting that case studies help researchers to develop a broader scope of data in which the cases are situated.

The decision to employ a case study as a practical approach for this research was based on more than a methodological deliberation. Several case study examples and models for teacher education can be found in pertinent literature. Still, few of them can be considered as a practical model (Rosenberg & Yates, 2007) or linked to a theoretical framework that examines the teaching knowledge (Tellis, 1997). Thus, the design of the present study was designed to adapt to the context of initial teacher education in Malawi, so the pre-service teachers’ understanding of teaching knowledge and learning experience for teaching mathematics in primary schools could be examined.

Malawian primary teacher education colleges offer a two-year course comprising six terms (as described in Chapter 3). During the first and second terms, pre-service teachers take theoretical classroom-based courses, whereas in the third and fourth terms, they go to local schools to complete supervised teaching practice. In the last term, they return to the college for more theoretical courses and reflection activities, allowing them to learn further from their teaching experience. To align with these distinct phases of the teacher education program, this research was conducted in three phases, denoted here as *moments*: Initial Moment (IM), Second Moment (SM), and Third Moment (TM).

4.1.1 The Initial Moment

During the IM, a first meeting was held with a group of twenty-three pre-service teachers from two beginning classes, who volunteered to participate in the study. During this meeting, all pre-service teachers completed a questionnaire comprising of both closed-ended and open-ended questions (see Appendix 1). These questions probed into the characteristics and profiles of the participants as well as their teaching experience (TE), mathematical interests in high school (MIHS), and mathematical interests for teaching during college (MIC). The pre-service teachers' profiles were arranged according to six categories that combined their TE, MISH, and MIC, as illustrated in Figure 3 and summarized in Table 3.

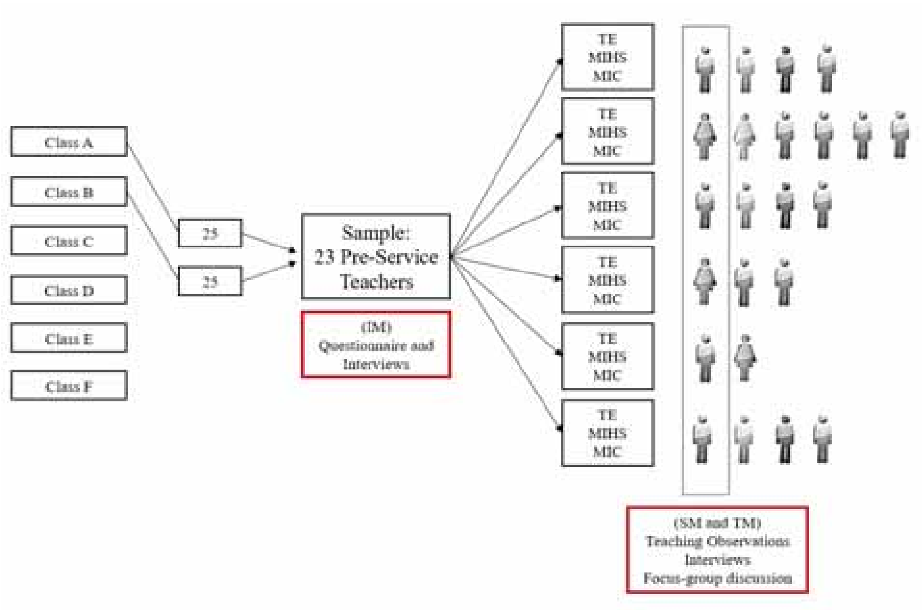


Figure 3. Number of participants during IM, and the selection process for the SM and TM.

The questionnaire also contained a five point Likert-type scale table (Krosnick, 1999), allowing the respondents to rate the relevance from *not at all important* to *extremely important* (see Appendix 1) of the knowledge and skills necessary to carry out tasks of teaching mathematics in Malawian primary schools. The list of specific knowledge types was created on the basis of the Ball et al. (2008) theory and related works, and it was a point of focus not only during IM but throughout SM and TM. The types of knowledge and skills the participants were required to rate included:

a) Knowledge of mathematical concepts contained in the school curriculum,
b) Knowledge of mathematical concepts outside (beyond those prescribed by) the school curriculum,
c) Using mathematical terms and notations correctly,

d) Ability to verify the accuracy of the mathematical definitions and examples in the textbooks,
e) Ability to identify student errors,
f) Ability to identify and explain where and how the students' errors occur,
g) Understanding the methods proposed by the textbooks to solve mathematical problems,
h) Knowledge of the different methods (generally not contained in the textbooks) that can be adopted to solve mathematical problems,
i) Ability to introduce mathematical concepts in exactly the same way as the textbook suggests,
j) Knowing how to introduce mathematical concepts and procedures in different ways (usually not contained in the textbooks),
k) Ability to prove mathematical statements,
l) Knowing how to give contextual examples for underlying mathematical ideas,
m) Knowledge of advanced mathematics,
n) Ability to engage students in the learning process,
o) Ability to explain how a particular mathematical topic is related to another mathematical topic,
p) Ability to explain how a mathematical topic is associated with other subject(s),
q) Knowing how to link mathematical topics to real-life situations,
r) Ability to organize activities outside the classroom,
s) Anticipating students' errors and common misconceptions,
t) Interpreting students' ideas correctly,
u) Ability to predict what students are likely to do when given specific tasks and what they will find interesting or challenging,
v) Knowing how to design a lesson effectively,
w) Ability to connect a lesson with another lesson,
x) Ability to choose appropriate exercises for instructional tasks,

y) Knowing how to use different teaching methods to teach mathematics,
z) Ability to evaluate the advantages and disadvantages of using specific representations,
aa) Comprehending the purposes and ethical values of education,
bb) Knowledge of the history of mathematics,
cc) Knowing how different cultures (or nations) develop and use mathematical ideas,
dd) Willingness to conduct research to uncover new facts and ideas related to the curriculum,
ee) Understanding what students find interesting and motivating in mathematics, and
ff) Knowing what students find easy and difficult in learning mathematics.

This list of items was based on a comprehensive literature review of studies about the knowledge teachers need to possess to teach mathematics (Ball et al., 2008; Jakobsen et al., 2012; Shulman, 1987). Each of these items was arranged according to one of the six knowledge domains suggested by Ball et al. (2008), as shown in Appendix 1, allowing connections to be made among the three study moments rather than imposing any particular domain on any research process. As an adaptable methodology was embraced for the present study, allowing items to be added or removed as the data was being collected, allowed each item to be seen as a preliminary code that contributed to the organization and framing of the item(s) according to the perspective of the pre-service teachers (as discussed further in Chapter 4.3 Data analysis).

After all the participants completed the questionnaire, each attended an individual interview which explored their responses in depth. During these interviews, the participants also discussed their views and ways of understanding the items listed in the questionnaire. The goal of the interviews was to determine the relevance that Malawian pre-service teachers assign to the knowledge and skills entailed in teaching (Ball et al., 2008) when it comes to teaching mathematics in primary schools. The pre-service teachers were also prompted to discuss their motives for and expectations of studying and becoming a primary school teacher. The interviews were video recorded during the process. This opening moment with the pre-service teachers was fundamental for determining the understanding that pre-service teachers have of the knowledge needed for teaching mathematics at the beginning of their teacher education.

4.1.2 The Second Moment

The Second Moment (SM) was based on *systematic observations* (Jablon, Dombro & Dichtelmiller, 2007) of the pre-service teachers' lessons on specific mathematical topics in local primary schools. This moment occurred during the in-field activities in local schools with a smaller sample of pre-service teachers than in the IM. Due to time and resource limitations, only one pre-service teacher per category was selected, resulting in observations of six pre-service teachers with different teaching experience and subject preference in high school and for teaching in college, as outlined in Table 3.

Table 3. Characteristics of the cases selected for the SM observations

Research sample selection criteria	**Pre-service teachers' anonymized names**
TE/MIHS/MIC	Martin
TE/No MIHS/No MIC	Mario
TE/No MIHS/MIC	Clara
TE/ No MIHS/MIC	Denise
No TE/ MIHS/MIC	Patrick
No TE/No MIHS/MIC	Daniel

TE, Teaching experience; MIHS, Mathematical interests in high school; MIC, Mathematical interests for teaching during college.

Table 3 shows the six types of profiles and the corresponding names given to the pre-service teachers. The first four pre-service teachers (Martin, Mario, Clara, and Denise) has previous experience in teaching. Martin is a 20-year-old, and he had one year of experience as a secondary teacher before entering the teacher education college. Martin expressed interest in mathematics during high school, as well as preferences for the mathematics courses at the teacher education college. On the other hand, Mario (22 years old), Clara (23 years old), and Denise (24 years old) demonstrated no interest in the mathematics subject during elementary school. Mario reported having no affection for teaching mathematics during college. While Mario and Denise both had one year of experience as primary school teachers, Clara had some experience as a home tutor in mathematics. At the bottom of Table 3, Patrick (23 years old) and Daniel (21 years old) presented no teaching experience prior to entering teacher college. While Patrick's favorite high school subjects were biology and mathematics, Daniel's was the English language.

The six pre-service teachers mentioned in Table 3, chosen among the twenty-three that took part in the IM, were selected based on three criteria that could reflect the diversity of the pre-service teachers' profile (Figure 2) and the feasibility of observing them during teaching practice in local schools. Although the six participants selected for SM were considered representative of the diversity of entering pre-service teachers' profiles in teacher education, the study also explored the cases in its singularity. According to Thomas (2011), "the subject (the case) is not selected based upon a representative sample, but rather is selected because it is interesting, unusual, striking, and may cause changes in the characteristics and specificities of the object" (p. 514).

Some of the Malawian primary schools in which pre-service teachers were required to complete their practice sessions were far from urban areas and thus difficult to access. As each pre-service teacher was located in a different school assigned to a different standard, the topics of their lessons varied. In addition, as each pre-service teacher had a different lesson schedule, this had to be considered when planning the observations and interviews.

During the systematic observations, focus was given to the ways pre-service teachers employed at least one of the items listed in the IM questionnaire during their teaching practice, such as answering students' questions, connecting prior knowledge to the new lesson topic, or using resources to introduce a mathematical concept. The goal of these observations was to capture specific situations and (verbal and non-verbal) behaviors in which the pre-service teachers employ the knowledge listed in IM to carry out tasks that emerge when they teach mathematics in the primary school. For one week, a mathematical lesson provided by each pre-service teacher was video-recorded without interference.

The establishment of the items listed in the IM questionnaire survey was crucial for conducting systematic observations of pre-service teachers' lessons and in guiding the individual post-lesson interviews. These interviews included questions regarding the tasks that the pre-service teachers faced during their teaching practice and the knowledge they drew upon to carry out these tasks. The discussions generated during these interviews revolved around the short videos of their lessons, which were presented to the pre-service teachers before asking them to elaborate on specific actions and the kind of knowledge used (or could have been used) to carry out the tasks in question effectively. This aspect of the study was essential to gain insight into the way Malawian pre-service teachers negotiate their understanding of the knowledge needed for teaching mathematics while conducting lessons.

4.1.3 The Third Moment

As noted previously, the TM occurred in the last terms of the program when the pre-service teachers came back to college to take additional courses and reflect upon their teaching experience. The data for this final moment of the study was gathered via a *focus group discussion* (Creswell, 2005; Wilkinson, 2004) that aimed to encourage the participants to articulate their way of understanding the teaching knowledge. These discussions covered the items listed in the IM questionnaire, focusing on the answers the pre-service teachers provided during IM and SM post-lesson interviews. The aim was not only to determine the understanding that pre-service teachers have of teaching knowledge at the end of the program but also to examine its evolution since the beginning of their teacher education. Table 4 details each of these moments.

Table 4. Summary of the empirical material collected in all three study moment

Moments throughout Teacher Education	Period	Objective	Instruments/Actions
Initial Moment (IM)	1 week during term 1 of the teacher education program	Obtain information of pre-service teachers' profile, motives and professional goals and find out their initial understanding of the knowledge necessary to teach mathematics in Malawian primary schools	Application of the questionnaire survey and like-scaled quiz about the relevance of the teachers' knowledge items proposed by the literature Semi-structured Individual Interviews about pre-service teachers' responses in the questionnaire survey

Second Moment (SM)	6 months during terms 3 and 4 of the teacher education program	Gather data on how pre-service teachers carry out tasks of teaching during practice and how they understand the knowledge that was used to carry out these tasks	Observe and video-record the pre-service teachers' mathematical lesson Interview the pre-service teachers after each lesson
Third Moment (TM)	2 days during term 6 of the teacher education program	Find evidence of the development of pre-service teachers' understanding of teaching knowledge during teacher education	Focus group discussion with all pre-service teachers about the evolution of their understanding of teaching knowledge during college

4.2 Ethics Issues

Prior to commencing the study, ethical approval was obtained from the Norwegian Social Science Data Service (NSD). The study had also been approved by the head of the teacher education college from which the participants were recruited and by the principals of the local schools where pre-service teachers completed their teaching practice. Each pre-service teacher also gave written consent for participating in the study, acknowledging that he/she could withdraw any time without restrictions or penalties.

As Malawian teacher education is under development, all ethical issues concerning the protection of the pre-service teachers' integrity had to be considered. One potential ethical issue pertained to recording lessons, individual interviews, and anonymization of the participants. As young adults have an awareness of the consequences of their choices in social life, they are a non-vulnerable group (Liamputtong, 2007). The present study considered the issues according to the NSD standards.

4.2.1 Acknowledgment for the Study

Prior to taking part in the present study, all pre-service teachers who agreed to participate received written and oral information about the project and signed an informed consent form. The information letter was previously accepted by the NSD and was written according to the NSD's guidelines under the conditions that the project end-date should be clearly stated to the participants and detailed information on safeguarding personal data (notification that data will be anonymized) would be provided.

Two pre-service teachers (one male and one female) asked whether the video recordings were going to be shared via YouTube, which must have seemed an exciting prospect for them. When they were told that the videos were going to be used for research purposes only, several individuals were clearly disappointed but still agreed to take part in the project. All pre-service teachers were informed that their names would be replaced by pseudonyms in all research materials. Even though the disappointment expressed by several pre-service teachers may have affected their motivation to participate in the study, data anonymization is strongly recommended by Biesta (2007) to avoid distraction and unusual behaviors during data collection, which might reduce the validity and credibility of the analysis (Allmark et al., 2009).

During the SM, care had to be taken to protect the integrity of the students, although their involvement in lessons was not the focus of the study. The local schools have an agreement with the teacher training college which allows research and experimental activities involving their pupils to take place. Nonetheless, prior to the present study, the college's head and all schools' headmasters received a letter explaining the study protocols. In addition, prior to classroom observations, both pre-service teachers and the researcher explained the study's objectives to the students, informing them about the presence of cameras in the classroom. This action was suggested by the headmasters of individual schools to ensure that the researcher's presence was relatively non-invasive and would not alter pupils' participation and performance during the lessons.

4.2.2 Issues Concerning Pre-service Teachers' Conduct

In all three study moments, pre-service teachers were interviewed regarding their views and understanding of the knowledge needed for teaching mathematics. Interviewing young adults presents specific challenges, such as *confidentiality* and *dual roles*, as articulated by Allmark et al. (2009) in a review article on research ethics of in-depth interviews. The interview subjects in this particular case were young adults who were expected to become professionals in teaching children. Hypothetically, this means that if a pre-service teacher participating in a research project displays a behavior, teaching practice, or opinion that goes against established norms for the profession, it could have negative work-related consequences and could be identified by colleagues or leaders in the final reports. Such legitimate concerns could preclude their participation in the project. Nevertheless, no misconducting behavior was observed, the focus (mathematical knowledge for teaching) was not particularly sensitive to the pre-service teachers, and they seemed pleased to contribute, so perhaps this issue is more of a challenge to the validity than it is to the ethics of the study.

4.3 Data Analysis

As all data gathered for this study was qualitative, it was subjected to *thematic analysis*. According to Braun and Clarke (2006), such an approach is a crucial tool for qualitative research since emerging themes allow researchers to examine "the ways in which events, realities, meanings, experiences and so on are the effects of a range of discourses operating within society" (p. 81). The thematic analysis enables researchers to identify and interpret important patterns or themes within the data to comprehend an issue, problem, or complex phenomenon (Vaismoradi, Bondas, & Turunen, 2013). Although criticized for its subjectivity and lack of transparency in the creation of themes (Attride-Stirling, 2001), the thematic analysis approach provides a rich and insightful understanding of the phenomena in focus since it can be applied across a range of theoretical and epistemological approaches, and used to expand or test a hypothesis (Braun & Clarke, 2006).

Thematic analysis is similar to content analysis (Javadi & Zarea, 2016; Vaismoradi et al., 2013), as the aim of both methods is to establish themes and categories. However, during content analysis, the frequency with which those categories appear in the data is also recorded (Bauer, 2000), whereas meaning of the data in context is the primary focus of thematic analysis. According to Joffe and Yardley (2004),

> the aspiration of thematic analysis, in particular, is to stay true to the raw data, and its meaning within a particular context of thoughts, rather than attaching too much importance to the frequency of codes which have been abstracted from their context (p. 3).

Thematic analysis in qualitative research can be both inductive and deductive. As Braun and Clarke (2006) explained, themes or patterns within data can be identified either in an inductive "bottom up" approach (Frith & Gleeson, 2004), or in a theoretical, deductive "top down" way (Boyatzis, 1998; Hayes, 1997). In either case, Braun and Clarke (2006) suggested six step-by-step guides for conducting thematic analysis that include:

1. Familiarization with the data,
2. Generating initial codes,
3. Searching for themes,
4. Reviewing themes,
5. Defining and naming themes, and
6. Producing the report.

These six steps served as a reference for the analytical approach adopted in this study. Data pertaining to all six pre-service teachers' lessons and interviews was transcribed using the NVivo software (QS International Pty Ltd., 2018) and was organized according to the items used in the IM questionnaire. These transcriptions were read and re-read against the original recordings for familiarization with the data and to ensure transcription accuracy. This data transcription and organization method was used in all three study moments (IM, SM, and TM), allowing the analysis to focus on specific characteristics of the teaching knowledge domains proposed by Ball et al. (2008).

Initial codes for the study were based on the list of knowledge and skills necessary to teach mathematics (Appendix 1) and the five categories for the concept of *understanding* pre-established on the conceptual framework (connectivity, sense-making, applicability, and justification), as illustrated in Chapter 2: Theoretical and Conceptual Background and Relevant Research. Figure 4 is an example of how the initial codes (Justification, Application…) were generated from the segments of the transcriptions and the conceptual frameworks' items.

MKT Domain	Knowledge Items (Initial Code)	Except from Mario's IM interview	Criteria for Understanding
CCK	Knowing mathematical concepts contained in the school curriculum	"For me, the content of the curriculum is important for teachers to know because it helps them to be better prepared to teach."	Justification
			Application

Figure 4. Organization of the data and generation of the initial codes within the conceptual framework.

The third phase of data analysis began with the elaboration of the themes. Each theme in this study was generated inductively to align with the research questions, rather than trying to fit it into a coding framework. The aim of this stage of the analytical process was to interpret the data to identify broader topics or headings applicable to the participants. According to Braun and Clarke (2006), this is a crucial stage of thematic analysis since it necessitates exploring the relationship among the codes and their different levels of connection. Some codes might be useful for building themes, while others might not; hence, it is the researcher's responsibility to combine, refine, or discard them to capture the contours of data.

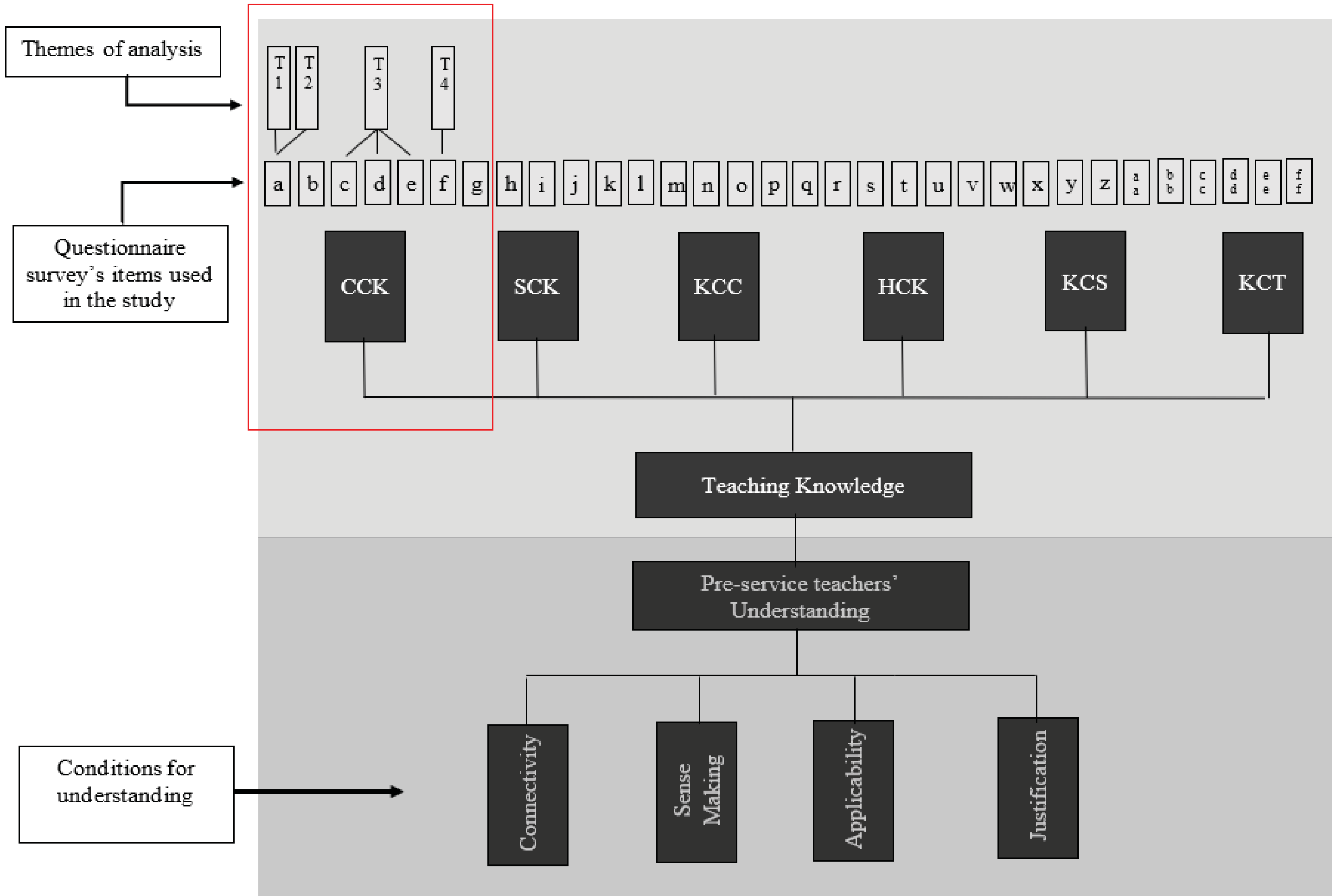
Themes are not a fixed category of analysis (Maguire & Delahunt, 2017). The redefinition of themes in the fourth phase of this study was shown to be "an iterative and reflective process that develops over time and involves a constant moving back and forward between phases" (Nowell, Norris, White, & Moules, 2017, p. 4). For instance, the theme *Knowledge of the mathematical concepts contained in the school curriculum* was initially generated from one item in the conceptual framework. However, as the analysis process developed after reading the data, this theme had to be divided into three sub-themes (importance for teachers to know mathematics for professional qualification, importance for teachers to know mathematics to solve problems in the classroom, and implications of school mathematical content for students learning of mathematics). Figure 5 shows how one of these themes (initial codes) was later modified and incorporated into a broader category denoted as *CCK for teaching and learning* (as discussed in the first journal article of this thesis).

MKT Domain	Knowledge Items (Initial Code)	Except from Denise's IM interview	Criteria for Understanding	Theme
CCK	Knowing mathematical concepts contained in the school curriculum	"For me, uh… it is important for the teacher to know the content because he or she can help the learners to understand when they are learning mathematics."	Justification	CCK for teaching and learning
			Application	

Figure 5. Production of themes from the initial codes and the conceptual framework.

Developing themes is one of the most complex phases of thematic analysis (Costa, Brenda, Pinho, Bakas, & Durão, 2016). DeSantis and Ugarriza (2000) opined that "a theme captures and unifies the nature of the basis of the experience into a meaningful whole" (p. 362). At this stage, the researcher refines and adjusts the themes to identify "the 'essence' of what each theme is about (as well as the themes overall) and determining what aspect of the data each theme captures" (p. 92). For each theme identified in this study, the aim was to find and construct a narrative that could apply to the entire data set and relate directly to the research questions (Braun & Clarke, 2006). This process was crucial for demarcating and organizing segments of similar or related text. Figure 6 shows how the themes of analysis were adjusted into the conceptual framework and the criteria for the concept of understanding.

Figure 6. Process of elaboration of the themes within the framework



After defining and refining the themes, Braun and Clarke (2006) suggest writing reports that link the themes back to research questions and pertinent literature. This last phase of the analysis of the data is reported in Chapters 5 and 6. In these chapters, after outlining the scope and content of the themes in each article, the connections and meanings between these themes were explored in light of the literature and the main research question of this study.

5 Results

This chapter presents the four articles chosen to compose this thesis with an emphasis on reviewing the findings in each article. As established in Chapter 1, the overall aim was to present and discuss the results of a study that sought to contribute to improving the quality and capacity of mathematics teacher education in Malawi. The main question of the study (How do pre-service teachers develop their understanding of the knowledge necessary to teach mathematics throughout teacher education?) was explored through three sub-research questions:

1. What understanding do pre-service teachers have of the knowledge needed to carry out the tasks of mathematics teaching at the beginning of their teacher education?

2. To what extent does the pre-service teachers' understanding of the knowledge needed to carry out tasks of teaching evolve through the discussion of practical experiences in college?

3. How do pre-service teachers develop their understanding of the knowledge needed to carry out the tasks of teaching throughout teacher training?

These three questions considered what prior research has uncovered regarding the knowledge needed to teach mathematics in primary school and how pre-service teachers acquire and construct an idea of this knowledge during teacher education.

This chapter is organized into three sections that summarize the results from each article chosen to compose the synopsis.

5.1 Article 1

Jacinto, E. J, & Jakobsen, A. (2020). Mathematical knowledge for teaching: How do primary pre-service teachers in Malawi understand it?, *African Journal of Research in Mathematics, Science and Technology Education*, *24*(1), 31─40. Doi: 10.1080/18117295.2020.1735673

This article examined the understanding that six pre-service teachers have of the mathematical knowledge, within the scope of subject matter knowledge domains (CCK, HCK, and SCK), they need to carry out their work as teachers of mathematics. This was addressed through the following research question: How do Malawian pre-service teachers understand the mathematical knowledge for teaching in primary schools? This question relates to the first sub-research question of this thesis: What understanding do pre-service teachers have of the knowledge needed to carry out the tasks of mathematics teaching at the beginning of their teacher education? This article explored themes of analysis belonged to the KCC, KST, and KCT domains Ball et al.'s (2008) mathematics knowledge for teaching framework (see Figure 1).

Data for this article was based on a questionnaire survey and individual interviews with six pre-service teachers at the beginning of their teacher education. Article 1 drew upon four themes of analysis: CCK for Teaching and Learning; Relating Knowledge of Out-of-Curriculum Content to HCK; The Importance of SCK in Interpreting Students' Errors and Capacities; and SCK: Stimulating Mathematics Learning Through Different Approaches. The first theme focused on the case of three out of six pre-service teachers (Daniel, Denise, and Clara) who revealed traces of understanding that valued the relevance and applicability of mathematical content contained in the school curriculum for primary school instruction. These traces drew upon the students' needs and specificities and teachers' confidence about teaching mathematics. Based on their initial understanding, in-depth knowledge of school mathematical concepts was considered crucial for teaching mathematical concepts in different forms and helping students to meet the curriculum requirements, according to pre-service teachers' views at the beginning of their college education. Moreover, pre-service teachers assumed that subject matter knowledge should be used to encourage students to apply knowledge of familiar topics while in pursuit of new, more complex topics; they valued the knowledge of progressive learning of the subject matter to make connections between mathematical concepts.

The second theme embodied facets of the HCK domain. Although scholars recognized the differences between HCK and knowledge of content and curriculum (Jakobsen et al., 2012), the six pre-service teachers involved in this study still referred to knowledge about content outside the curriculum as a curricular horizon rather than as a mathematical horizon. The relevance of this type of knowledge, however, varied among the six pre-service teachers. Four pre-service teachers placed knowledge beyond the school's curriculum as less important—arguing that the Malawian government determines what needs to be taught in classrooms and that teachers do not need to go beyond the curriculum content. The other two pre-service teachers, however, demonstrated a more balanced point of view, contending that teachers need to possess knowledge outside the school's curriculum to handle the tasks of teaching. Some examples were given related to Ball et al.'s (2008) lists of tasks of teaching, such as properly answering students' questions, making content easier for students to learn, and connecting topics being taught with topics from prior or future years. Additional examples involved reflecting on what they are teaching and how they are sequencing the content of the curriculum, a necessary element of effective teaching. Additional findings revealed that the pre-service teachers acknowledge that content outside of the school curriculum can be essential, but it can be challenging for teachers in Malawi to access it, due to the work conditions and limited resources.

The third and fourth themes relate to the pre-service teachers' understanding of the knowledge needed within the domain of SCK. Whereas the third theme focuses on the knowledge needed to interpret students' errors and capacities, the fourth covers the knowledge necessary to carry out different approaches in the classroom. The themes suggested that at the beginning of their training, the pre-service teachers revealed a reasonable understanding of why and how students make mistakes and what knowledge and skills can be employed to minimize these mistakes. Martin, Patrick, and Denise backed up their arguments by describing the implications of defining how a teacher should teach in Malawi. For them, the knowledge necessary to recognize student errors and capacities helps teachers understand students' thinking and make effective decisions to improve their education. Daniel, Clara, and Mario, in contrast, supported their claims with different reasoning. They concluded that one of the crucial attributes of such knowledge is the ability to guide students to discover the nature of their problems themselves. These pre-service teachers also revealed an awareness of the applications of the knowledge of students' abilities in helping teachers not only identify children's inaccuracies or errors but also assist them to understand why these problems occur and what can be done to overcome them.

5.2 Article 2

Jacinto, E. L., & Jakobsen, A. (2019). Pedagogical content knowledge for teaching mathematics: What matters for pre-service primary teachers in Malawi? In M. Graven, H. Venkat, A. Essien, & P. Vale (Eds.), *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education: Research Reports* (Vol. 2, pp.424─431). Pretoria, South Africa: PME.

This article focuses on the understanding that three pre-service teachers (Martin, Carlos, and Clara) had of the knowledge needed to teach mathematics within the realm of pedagogical content knowledge.

This was addressed through the following research question: how do pre-service teachers perceive the pedagogical content knowledge for teaching mathematics? This question is also linked to the first sub-research questions of this thesis as it provides insights into what understanding pre-service teachers have of the knowledge needed to carry out the tasks of mathematics teaching at the beginning of their teacher education. Article 2 explored themes of analysis covering the domains of KCC, KST, and KCT, one side of the framework model of mathematics knowledge for teaching provided by Ball et al. (2008; see Figure 1).

As explained in the methodology chapter, these themes of the overall work were initially developed from the list of items in the questionnaire survey given at the beginning of the study. Later, the themes were revised and adjusted to the forms of knowledge and skills pre-service teachers revealed when reflecting on practical situations of teaching. Thus, for this article, the themes of analysis were decision-making in teachers' KCT, relations between KCT and KCC, and adaptations of the classroom activities from students' contributions and levels of understanding.

The first theme, decision-making in teachers' KCT, was examined through the knowledge needed to decide on the best examples and representations to use for given instructional objectives. Findings from Martin's case revealed that, at the beginning of teacher education, he recognized that decision making was an integral part of a primary school teacher's skill management for teaching, an idea that resembles KCT in terms of the knowledge of strategies and representations for teaching particular topics. For Martin, the knowledge of instructional practice design combines the knowledge not only of the organization of the lessons but also how to guide mathematical tasks in classrooms gradually, according to students' capacities for learning. Martin's views, however, revealed that the context in which such knowledge applies is a unique characteristic of teachers' pedagogical knowledge for teaching in crowded classrooms. For Martin, teaching mathematics under such circumstances should not be based solely on manuals, guidelines, and textbooks; the teacher's pedagogical skills should be focused on the general organization of the lessons and the conditions and possibilities given to the students to be familiar with the content.

The second theme, relations between KCT and KCC, comprised analyses of Carlo's perception of the pedagogical content knowledge for teaching mathematics. Such perception is framed by aspects of the knowledge that integrated elements from both KCC and KCT domains. Findings show that one crucial aspect of teaching mathematics effectively is the condition of being resourceful. For Carlos, effective teachers must possess appropriate knowledge of how to select, revise, and use appropriate didactical materials to facilitate student learning of mathematics. By developing the capacity to discern the advantages and disadvantages of using locally available resources, teachers in Malawi can create learning situations that are more accessible to the students, Carlos assumed. Such knowledge not only helps Malawian primary teachers to become more familiar with the tools needed for mathematical lessons but also improves their understanding of how simple objects can produce the need to learn mathematics in students.

The last theme explored the perceptions of Clara regarding features of KCS and knowledge about creating classroom activities based on students' contributions and level of learning. The findings revealed a significant aspect of teachers' KCS. Although students' ideas should be considered in the classroom, methods proposed by the teacher are still dominant. For Clara, the knowledge of the students' capacities can help teachers better understand and react to students' ideas while teaching, but the use of multiple approaches might not be helpful for those who are still learning the content. According to Ball et al. (2008), anticipating what students are likely to think and what they may find confusing is an essential skill for helping teachers balance and adapt their work according to students' contributions and level of learning. Moura

(2010) reminds us that every idea exchanged is valuable for improving the quality of a teacher's work. By knowing and interacting with their students, teachers can expand their conceptions and learn new ways to act that would promote the development of learners' reasoning abilities (Moura, 2010).

This article concluded that pre-service teachers in Malawi acknowledged characteristics of pedagogical content knowledge that are pertinent for satisfying students' contextual needs. Although pedagogical content knowledge has assumed different forms of conceptualization in the literature, this article showed that pre-service teachers in Malawi can provide unique insights about the conceptualization of pedagogical content knowledge in the teaching of mathematics.

5.3 Article 3

Jacinto, E. J., Jakobsen, A., & Bjuland, R. (2020). Understanding of the knowledge necessary to sequence tasks in mathematical instruction: The case of Malawian pre-service teachers, *International Journal of Science and Mathematics Education*. (Accepted for revision, resubmitted)

This article examined the understanding that two pre-service teachers (Denise and Martin) had of the SCK when discussing knowledge demands for sequencing instructional tasks in mathematics. This was addressed through the research question: how do Malawian pre-service teachers develop their understanding of the knowledge needed for sequencing tasks for mathematical instruction? This question relates to the second sub-research question of the overall work (To what extent does the pre-service teachers' understanding of the knowledge needed to carry out tasks of teaching evolve through the discussion of practical experiences in college?). The articles' question focuses on the knowledge needed to carry out one task of teaching (sequencing tasks for students during mathematics instruction) that is part of the SCK domain (Ball et al., 2008).

In this article, sequencing tasks were described as problems or activities that (pre-service) teachers developed and posed to students. The cases of Denise and Martin were worthy of investigation because, at the start of their teacher education, the questionnaire/interviews revealed differences in their teaching experience prior to starting teacher education. It also showed they had different subject preferences during high school and college but similar views on the importance of the knowledge of sequencing tasks for teaching mathematics. They also followed distinct and complementary paths of development regarding their understanding of specialized content knowledge.

Article 3 combined data from two moments in the study that occurred during the theoretical course at the TTC and supervised practice in local schools. In this second moment, two pre-service teachers (Denise and Martin) revealed a common understanding of the knowledge related to content development in practical situations. For them, something that makes teaching mathematics unique is knowing how to introduce mathematical concepts gradually, from simple to complex. Denise pointed out that the teacher should aim to ensure students' inclusion by making the content more comprehensive and useful. Such a process should be considered by the teacher so the students can follow the progression of the content and make sense of it from different perspectives. In contrast, Martin described the knowledge necessary to sequence tasks in mathematics instruction based on the demands of the school curriculum. In his view, the teachers should know how to introduce concepts and examples from simple to complex, according to textbook standards. It is also worth noting that both Denise and Martin displayed practical understanding in the earliest stage of the study by accomplishing task progression in mathematics as a way to achieve a twofold teaching goal: (1) the promotion of students' confidence and ability to solve mathematical problems and (2) alignment with the school textbook's standards.

The understandings Denise and Martin initially showed took different forms after their teaching practice in local primary schools. When teaching multiplication, Denise tended to value the order of representations when fostering students' mathematical learning. Although she knew multiplication can be commutative—that is, numbers may be multiplied together in any order—she believed it was crucial to mainly teach students the positional meaning posed by the coefficients rather than show them that numbers could be multiplied together in any order. It is important for teachers to have such knowledge, but there is a risk that it can confuse students and lead to instructional misdirection with the textbook guidelines. Denise believed that by knowing and teaching multiplication in a manner that students can follow gradually, teachers can help students develop generalizations about the content. This knowledge and belief provided a better picture of how Denise understood the knowledge needed to sequence tasks in the teaching of multiplication.

Martin's case was analyzed in relation to the knowledge necessary to sequence tasks for teaching fractions. When reflecting on a situation that demands a type of knowledge that helps students to develop fraction concepts through geometrical representation, Martin recognized the limitations of using only the curriculum to prepare mathematical lessons. The findings suggested a new understanding of what the work of teaching entails in Martin's case. At this stage, Martin acknowledged that although knowledge of content and curriculum is important, a teacher also needs to develop a sense of how students can interpret connections between tasks and the ascendant progression of concepts from simple to complex. Martin argued that the teacher plays a significant role in organizing more accessible and connective tasks to allow students to solve mathematical problems posed in the classroom. An additional finding that revealed traces of Martin's understanding of the knowledge necessary for sequencing tasks is that teachers need to possess a high degree of coherence and amplitude to teach mathematical contents progressively. When discussing ways of teaching fractions using geometrical representation, Martin asserted that a particular characteristic of an effective teacher is the autonomy to choose and modify tasks that are both easier and harder for the students, using knowledge they already possess. This crucial feature showed an evolving form of understanding.

5.4 Article 4

> Jacinto, E. J. (2020). An analysis of pre-service teachers' understanding of the knowledge entailed in the work of teaching: Insights from Malawi. *Journal of Mathematics Teacher Education.* (Under review)

This article 4 was thought to answer the third sub-research question: How do pre-service teachers develop their understanding of the knowledge needed to carry out tasks of mathematics teaching throughout teacher training? The article addressed this question with a particular focus on the knowledge needed to organize instructional tasks progressively to help students solve mathematics problems and to use locally available resources to create multiple representations in mathematics. The data combined responses from the questionnaire surveys and individual interviews at the beginning of their teacher education, lesson observations and post-lesson interviews during teaching practice, and focus group discussion when the pre-service teachers were back at the college after teaching practice.

Article 4 focused on the case of three pre-service teachers (Denise, Martin, and Mario). Whereas Denise's and Martin's cases were studied under the theme Knowledge of Instructional Task Progression to Help Students Solve Mathematical Problems, Mario's case was examined under the theme Ability to Use Locally Available Resources to Create Multiple Representations. These two themes were part of the domain of SCK and were developed following the same process used in Articles 1, 2, and 3 (see Chapter 4, Methodology). Table 5 shows how data from Article 4 covered the SCK domain and sample subjects of the study.

Findings from the first theme—Knowledge of Instructional Task Progression to Help Students Solve Mathematical Problems—suggested that the pre-service teachers acknowledged the relevance of and need for the knowledge required to organize instructional tasks according to students' needs and curricular demands. In the case of Denise, such knowledge manifests as systematic expertise with the emphasis on proceeding in small steps and checking students' understanding, with the goal of promoting their problem-solving skills and reasoning. As was the case at the beginning of the study, Denise revealed a broader understanding of the knowledge needed to carry out task progression, but during the teaching practice, she expanded that the understanding by considering knowledge not only of differences between simple and complex content but also about the ways learners make sense of the content logically and critically during the lessons. Such a course of understanding seemed to have matured over time. During the focus group discussion, Denise expanded her thoughts when describing that the organization of tasks for students should be seen by the teacher not only in parts but as a whole when solving mathematical exercises.

In Martin's case, his understanding of task progression developed over the course of the teacher education, but in a different direction. In his first interview, as part of the first moment of the study, he presented an inadequate understanding by arguing there was no need for teachers to know much about how to introduce content, given that textbooks and curriculum guidelines are already recommended to be followed. However, Martin's view evolved as he gained practical teaching experience in local schools. By recognizing the limitations of the teacher's textbook in suggesting instructional tasks and examples for students, Martin allotted more value to teachers' autonomy and students' capacities for learning. Then, at the end of his teaching education, he sustained such thoughts by justifying the idea that teachers must identify and predict any problem students might have during mathematics lessons in addition to helping them comprehend the similarities and differences among examples within those lessons. Martin also opined that teachers need to use their knowledge of content and students to decide how to organize and present tasks in the classroom. In this regard, data show that by the end of the teacher education, Martin's understanding of the knowledge needed to carry out instructional tasks became more closely related to the concept of SCK provided in the literature.

Findings from the second theme of analysis—Ability to Use Locally Available Resources to Create Multiple Representations—suggested that pre-service teachers can develop a meaningful understanding of the tasks and knowledge necessary to carry out these tasks properly in the Malawian context. The case study in this theme featured Mario, a pre-service teacher with teaching experience but no interest in expanding his mathematical knowledge.

During the first moments of the study, Mario expressed an intuitive interpretation of the knowledge needed for teaching using mathematical representations. His argument was that teachers must think of diverse ways of teaching a concept to provide multiple opportunities for students to learn mathematics. The use of locally available resources was described as crucial for teachers to introduce mathematical concepts better and to propose mathematical problems to students. In Mario's initial understanding, creating opportunities for students to interpret mathematical concepts through representations seemed to play a crucial part in teaching knowledge. During teaching practice, Mario's understanding of the knowledge needed to represent mathematical ideas expanded to include a new aspect: the connections between mathematical representations. This is a special component of mathematics teaching; concepts and procedures are enhanced in teaching to help students consider not only concrete but also pictorial and abstract representations. In the last moment of the study, Mario demonstrated an evolving understanding of the knowledge needed to carry out multiple representations in the classroom. For Mario, teachers not only need to be familiar with multiple representations but also need to be able to structure representations, understand the connections between them, and know how to help students gradually interpret and move between representations.

Mario's understanding of the knowledge of mathematical representation also seems to be driven by the specific purposes of promoting generalizations: one crucial aspect when teaching mathematics through representations is to focus on the meaning of mathematical concepts so students can understand and apply them in different situations. According to Mario, such knowledge allows teachers to build bridges from their personal representations to more realistic and useful ones, contributing to a better understanding of the teaching and learning of mathematics.

5.5 Summary of the Findings across Articles

The findings of the four articles presented in this chapter comprised a set of themes that helped to situate the thesis' object of study (the understanding pre-service teachers developed of the knowledge necessary to teach mathematics throughout teacher education). Collectively, the articles revealed the evolving facets of pre-service teachers' understandings within a respective type of knowledge needed for teaching mathematics. In summary, the articles provided insights that refined the main research question over the span of the following themes of analysis:

1. CCK for teaching and learning,
2. Relating knowledge of out-of-curriculum content to HCK,
3. The importance of SCK in interpreting students' errors and capacities,

4. SCK: Stimulating mathematics learning through different approaches,
5. Decision-making in teachers' KCT,
6. Relations between KCT and KCC,
7. Adaptations of the classroom activities from students' contributions and levels of understanding,
8. Knowledge of how to sequence instructional tasks to foster students' learning of mathematics,
9. Knowledge of instructional task progression to help students solve mathematical problems,
10. Ability to use locally, available resources to create multiple representations.

Although referring to particular domains of mathematical knowledge for teaching, each of these themes contributed to answering the research questions. Again, these 10 themes of analysis are a refinement of types of knowledge initially designed for the study. Table 5 locates each theme according to the specific research question and the main research questions. The 'Th' inserted in the cells represents one of the 10 themes listed above.

Table 5. Summary of the themes according to the cases and research questions

Cases	Overall Research Question: How do pre-service teachers develop their understanding of the knowledge necessary to teach mathematics throughout teacher education?																	
	Question I (First Moment): What understanding do pre-service teachers have of the knowledge needed to carry out tasks of mathematics teaching?						Question II (Second Moment): To what extend the pre-service teachers' understanding of the knowledge needed to carry out tasks of teaching evolve when discuss practical experiences in college?						Question III (Third Moment): How do pre-service teachers develop their understanding of the knowledge needed to carry out tasks of teaching throughout teacher training?					
	SMK			PCK			SMK			PCK			SMK			PCK		
	CCK	HCK	SCK	KCC	KCS	KCT	CCK	HCK	SCK	KCC	KCS	KCT	CCK	HCK	SCK	KCC	KCS	KCT
Martin	Th1	Th2	Th3			Th5			Th8						Th9			
Mario		Th2	Th3 Th4												Th10			
Patrick		Th2																
Daniel	Th1		Th3			Th6												
Denise	Th1	Th2	Th3 Th4						Th8						Th9			
Clara	Th1	Th2	Th3			Th7												
	Article 1					Article 2			Article 3						Article 4			

6 Discussion

This chapter aims to critically reflect on the key results presented in the previous chapter in relation to the findings yielded by other studies and to the theory on which this study is grounded. The chapter's content is cleaved into three sections, aligning the questions addressed in the four articles (Chapter 5) with the three sub-research questions of this thesis (Chapter 1). Findings from the data and analysis reported in the articles are also discussed in relation to the overall research question (Chapter 1) and a broader perspective linked to the literature, and previous research in the field (Chapter 2).

The overarching goal of the research reported in this thesis was to investigate how pre-service teachers develop their understanding of the knowledge necessary to teach mathematics as they progress through teacher education. The data analysis was conducted on the premise that effective teaching and extensive subject matter knowledge require a complete and genuine understanding of the knowledge and skills entailed in the practical work of teaching (Ball, 2017). Although the findings presented in Chapter 5 provided answers to the research questions, further elaboration is needed to draw meaningful conclusions and identify their implications for research and practice. Hence, in the sections that follow, the findings are discussed in relation to each sub-research question and the overall goal of this study.

6.1 Pre-Service Teachers' Initial Understanding of the Knowledge Needed to Carry out Tasks of Mathematics Teaching

Effective teaching often involves using the knowledge and skills required for the tasks of teaching (Ball & Forzani, 2009). As the ability to teach effectively demands much more than subject matter expertise

(Ball et al., 2008), pre-service teachers are required to develop specialized knowledge that helps them not only engage students in the learning process but also meet the demands and challenges of society (Day, 2002; Darling-Hammond, 2006).

According to the Malawian IPTE curriculum, primary pre-service teachers are expected to become autonomous agents and lifelong learners (see Chapter 3). However, the steps taken to attain this ambitious goal are not clearly delineated in this curriculum (Malawian Institute of Education, 2017). In this context, pre-service teachers seemed to be under considerable pressure to meet the requirements of the teacher education program which might be a factor that influences the pre-service teachers learning (Salagi, 2018). As the findings in this thesis indicate, most pre-service teachers who took part in the present study demonstrated partial understanding of the tasks involved in the work of teaching at the beginning of teacher education; this aligned with Barkatsas and Malone's (2005) report on pre-service teachers' beliefs regarding the nature of mathematics knowledge. At the beginning of teacher education, most pre-service teachers could identify the core tasks of teaching that are characteristic of the Malawian context as noted by Jakobsen et al. (2017), even though they had difficulties in providing examples and making connections between students' needs and curricular demands. The current study showed that at the beginning of teacher education, pre-service teachers' understanding of teaching knowledge was mostly rooted in school curricula and IPTE program guidelines without further explanation of the origin of these sources.

One reason why the pre-service teachers demonstrated such partial understanding of the knowledge teaching during the early stages of teacher education might be linked to the fact that they possessed different views of teaching and diverse motives for entering teacher education programs (Bergmark, Lundström, Manderstedt, & Palo 2018; Watt & Richardson, 2008). These factors might have substantial implications for pre-service teachers' way of understanding the work of teaching and the skills necessary to conduct it effectively (Zhang, Hawk, Zhang, & Zhao, 2016). These views seem to apply to the Malawian context, given that the results reported in Article 2 indicate that pre-service teachers have diverse educational backgrounds and expectations of the teaching profession. Nonetheless, the overall study indicates that most pre-service teachers expressed a strong motivation for covering the fundamental values stated in the Malawian school curriculum and understood the knowledge and skills necessary to perform these tasks as a primary source for teaching mathematics in primary school.

The initial understanding of teaching knowledge revealed by the pre-service teachers was also spontaneous. The analysis aligns with studies that suggest pre-service teachers appear to focus almost exclusively on the knowledge of the content rather than the knowledge of interpreting students' errors and capacities (Cady & Rearden, 2007; Marshman & Porter, 2013). Pre-service teachers tended to elicit events from practices they have experienced or observed in elementary schools to justify the necessary knowledge for primary school teachers in Malawi to possess (Articles 1 and 2). However, there was no evidence that pre-service teachers were aware of important structures of the mathematical concepts (see Hill, Sleep, Lewis, & Ball, 2007) and the knowledge that goes beyond school content (Jakobsen et al., 2012; Wasserman & Stockton, 2013). Thus, it is not easy to assume how pre-service teachers will employ teaching knowledge in practice from their initial understanding at the beginning of teacher education (Waters-Adams, 2006).

The pre-service teachers who focused on aspects of specialized content knowledge revealed a concern about classroom management. For instance, in Article 1, the pre-service teachers, Mario and Denise, acknowledged the relevance for teachers to create a learning environment for all, using locally available resources. Still, there is a need for awareness of not only what and when specific didactical materials should be employed but also how these materials can help the teachers carry out the tasks of teaching effectively. This argument is supported by O' Neill and Stephenson (2012), and also related to the idea that at the beginning of teacher education, pre-service teachers can demonstrate an advancing form of understanding of the teachers' responsibilities and the tasks that involved different approaches to help students to learn properly (Ball, 2017; Ball & Forzani, 2009; Lauerman & Karabenick, 2013; Mitchell et al., 2014).

6.2 Resignification of Pre-Service Teachers' Understanding of the Knowledge Needed to Carry Out Tasks of Mathematics Teaching

As a result of the coursework in college and teaching practices in local schools, pre-service teachers presented a more complex view in their understanding of the knowledge needed to carry the tasks of teaching mathematics effectively. During the teaching practice, the pre-service teachers found they had to deal with many of the tasks of teaching mathematics. After discussing some of these tasks, the pre-service teachers presented a deeper understanding of the knowledge needed to sequence tasks for mathematics instruction. They were able to provide alternative examples and approaches to engage students and instructional suggestions for helping students overcome errors in solving mathematical problems. For some researchers, other than the fact that previous experiences and ideas about teaching have a strong influence in the way pre-service teachers understand teaching, a critical factor that might contribute to its restructuring is instructional practice (Charalambous & Litke, 2018; Osmanoglu, 2016; Ulusoy, 2020; Yang, Kaiser, König, Blömeke, 2020).

The findings from the current study suggest that pre-service teachers tend to pay attention to both specific and general aspects of teaching rather than only focus on general tasks or events, as also found by Star and Strickland (2008). This result is consistent with those by Chamoso, Cáceresa and Azcárateb (2012) and Snyder (2010) who observed that pre-service teachers could demonstrate an evolving understanding of the tasks and knowledge entailed in the work of teaching when discussing their actions, decisions, interactions with the students, and responses to situations that emerged during the teaching that took place during their teaching practice.

One of the teaching task characteristics of mathematics teachers in Malawi is the ability to create their own didactical materials for teaching (Kazima et al., 2016). Although such a task is not part of Ball et al.'s (2008) conceptualization of mathematical tasks of teaching, using teaching aids from local resources is one of the tasks that pre-service teachers are required to learn to carry out when they are in college. Grossen (1996) evaluates this demand by pondering the teachers' real function. The author further states, "to ask that teachers create all of their own tools and curricula is like asking doctors to invent all of their own drugs; like asking airplane pilots to build their own airplanes. When would teachers have time to do this?" (p. 20). For the pre-service teachers who participated in this study, it was determined that although the tasks of creating their own teaching resources can be demanding, it was also satisfying (Article 3). By creating their own materials, Freire (1996) argues that pre-service teachers are able to know their own knowledge and abilities (financially and mentally) to search, create and transform the locally available resources into learning mediation tools for the students' learning. Therefore, a teacher needs to be flexible, creative, and (s)he needs to have an explicit understanding of the nature of the taught mathematical content and the students' capacity to acquire such content (Article 4).

Another insight revealed in Article 3 was that pre-service teachers similarly recognized the need to organize instructional tasks to help students learn mathematics but from different perspectives. In Article 3, some pre-service teachers acknowledged that teachers should progressively introduce mathematical concepts from the simple to the more complex, as this accomplished a twofold teaching goal: (1) development of students' self-confidence in their ability to solve mathematical problems, and (2) alignment with the school textbook's standards. These findings are in alignment with extant research on teacher education that highlights the significance of teaching experience to shape pre-service teachers' conceptions of crucial aspects of the work of teaching (Hiebert, Morris, Berk, & Jansen, 2007; Hill et al., 2008; Jones & Vesilind, 1996).

According to Ryan, Carrington, Selva, and Healy (2009), teaching experiences in real schools are necessary for expanding pre-service teachers' views of the complexity and dynamics of teachers' work. During teacher education, pre-service teachers must be given opportunities to connecting theory and practice since this prompts them to reconsider their initial conceptions of the work of teaching mathematics and to make strategic decisions based on critical reflection (Sheridan, 2016). Pre-service teachers' understanding of teaching knowledge is influenced by their preconceptions or knowledge involved in the task of teaching and the quality of the personal experiences and theoretical guidance they receive during teacher education (Darling-Hammond & Youngs, 2002). This was particularly evident for pre-service teachers during teaching practice (Articles 3 and 4), whose understanding of the knowledge entailed in the work of teaching were initially aligned with the recommendations given in the teachers' guide and the students' learning needs.

However, pre-service teachers' understanding of teaching knowledge gradually evolved, primarily due to their ability to develop basic generalizations and connections between the mathematical content properties. This understanding seems to play a crucial role in the pre-service teachers' learning since it provides them with opportunities to make connections and build self-confidence in their ability to provide effective instruction (Darling-Hammond & Bransford, 2005; Le Cornu

& Ewing, 2008; Starkey, 2010). Consequently, the findings reported in this work shows how pre-service teachers evolve their understanding of the knowledge demands and the tasks of teaching when reflecting on their practical teaching experiences.

6.3 Development of the Pre-Service Teachers' Understanding of the Knowledge Necessary to Carry Out Tasks of mathematics Teaching

Results from Articles 1 and 2 align with the findings reported by Barkatsas and Malone (2005). These authors found that pre-service teachers begin teacher education with the beliefs and conceptions about teaching that are largely based on their own school experiences. At this stage of teacher education, pre-service teachers tend to demonstrate awareness of the common problems and tasks that teachers face in classrooms (Ball, 1990; Pajares, 1992) and the role teachers play in the students' learning (Enyedy, Goldberg, & Welsh, 2005). However, as evident from the findings presented in Articles 3 and 4, the pre-service teachers' views of what teaching entails might also continue to be influenced by standardized national curriculum and school textbooks.

Although research shows that pre-service teachers might present critical and open ideas about the work of teaching at the beginning of teacher education (Fajet, Bello, Leftwich, Mesler, & Shaver, 2005; Jao, 2017), only when discussing broader aspects of the work of teaching, the pre-service teachers were capable of justifying and making sense of the knowledge necessary to carry out the tasks of teaching mathematics (Article 1). In this regard, Letwinsky and Cavender (2018) and Quick and Siebörger (2005) clarify that when entering teacher education, pre-service teachers often base their views on teachers' guidelines and approaches on former successful practices, which might lead to a fragmented view and understanding of crucial aspects necessary for conducting teaching effectively.

In Malawi, research has indicated that primary pre-service teachers struggle to develop proper mathematical knowledge for teaching during teacher education (Jakobsen et al., 2018; Kasoka et al., 2017). This study shows that although pre-service teachers might hold partial and intuitive ideas of the work of teaching on entering teacher education, they can create different interpretations of the tasks of teaching and present evolving forms of understanding of the knowledge needed to carry out these tasks as they progress through teacher education. According to Gardner and Williamson (2007), this is a crucial component that might explain why pre-service teachers often struggle to reach an adequate level of knowledge and skills regarding the work of teaching constituents. During teacher education, the pre-service teachers are still searching for meanings and ways to make sense of what is involved in the work of mathematics teaching (Articles 1, 2, and 4), which might indicate the degree to which they value certain knowledge to carry out teaching tasks (Pillen, Beijaard, & Brok, 2012).

Although in the current study pre-service teachers' understanding of teaching knowledge is directed towards conventional teaching approaches, findings in the Articles 3 and 4 corroborate findings reported in Sheridan's (2016) work that shows that the different forms of understanding and views pre-service teachers hold during their teacher education can evolve with incorporation of new features as they gradually mature and experience new ideas or situations. For example, as demonstrated in Articles 3 and 4, teaching practice considerably enabled the pre-service teachers to reconsider their prior knowledge and understanding of teaching and the knowledge needed for teaching mathematics, reinforcing Smith and Lev-Ari's (2005) insights into the role of a practicum in pre-service teacher education. Thus, it is assumed that pre-service teachers' views and understandings of teaching knowledge evolve throughout teacher education as they began to comprehend the dynamics and complexities of the tasks involved in the work of teaching (Ball & Forzani, 2019).

A foremost component of teaching knowledge explored in this thesis was the tasks and knowledge related to the SCK domain. Although the IPTE program curriculum does not provide explicit details of the tasks of teaching within this domain (Malawian Institute of Education, 2017), the current study shows that the pre-service teachers tended to think of the knowledge for carrying out tasks of teaching not as a mere list of competencies or techniques for teaching (as indicated in Article 1), but rather as generic skills that help teachers present the content in a way that makes it more accessible and useful to the students (Article 1).

SCK is a domain that clearly connects with the knowledge teachers need to do the work of teaching mathematics in the classroom, and an understanding of the tasks of teaching that pertain to such a domain is highly relevant for pre-service teachers to make sense of teaching and different approaches to make the content more comprehensible for students (Ball et al., 2008). However, as the concept of SCK is relatively new to the field, more research is needed to steer how pre-service teachers can better acquire and understand the nature of the knowledge so they can respond to the everyday tasks of teaching, especially the tasks of teaching in challenging contexts.

It is also noteworthy that the pre-service teachers who took part in this study demonstrated the ability to adapt and use locally available resources to foster students' learning of mathematics. For instance, as indicated in Article 4, Mario, one of the participating pre-service teachers, realized from the practicum that the knowledge needed to teach mathematics also involved using different representations. The literature also contains studies showing that throughout teacher education pre-service teachers can become more aware of the tasks of teaching and the knowledge necessary to carry out these tasks if they are given the chance to explore and reflect on the teaching situations (e.g., Lauerman & Karabenick, 2013; Mitchell et al., 2014) By the end of teacher education, Mario's understanding of teaching knowledge involved a different facet from being familiar with multiple representations to being able to structure representations to understand the connections between them, and know how to help students gradually interpret and move between representations (Article 4). According to Pape and Tchoshanov (2001) and Tripathi (2008), these dimensions refer to the production of meaning regarding mathematical representations, an ability that Ball et al. (2008) have suggested is unique to the work of mathematics teacher.

Thus, based on the findings yielded by the present work, it can be argued that the development of SCK is critically important in teacher education since it can help pre-service teachers recognize and employ central aspects of effective teaching in practice. Various studies have also found that developing a deep understanding of SCK components is an important foundation for becoming a qualified teacher and capable of assisting students to learn mathematics (Chinnappan & White, 2015; Ding, 2013; Ndlovu et al., 2017). Moreover, by analyzing Mario's case from the perspective of the SCK domain—the knowledge of multiple representations to teach mathematics, in particular—it can be argued that pre-service teachers can acquire and deepen an understanding of the multifaceted nature of SCK and its role in the contexts of challenging teaching. Such a view is also shared by Provost (2013). This thesis complements these thoughts by supporting the view that the pre-service teachers' understanding of knowledge necessary for teaching is likely to evolve in different ways as they progress during teacher education. This peculiarity tends to help them get a sense of the theoretical landscape of effective teaching and the knowledge and skills that will better prepare them to teach mathematics (Ball, 2017; Ball & Forzani, 2009).

7 Conclusion and Implications

This chapter presents the thesis conclusion, a crucial component of the overall work. In this chapter, I sum up the most important insights, stress the limitations, and highlight the implications for further study.

As argued in Chapter 2, pre-service teachers have conceptions and beliefs about several different aspects of education. These conceptions and beliefs can have a significant influence not only in their learning process during teacher education but also in their future profession as teachers (Patrick & Pintrich, 2001). In this thesis, I focused on the understanding primary pre-service teachers develop of the knowledge necessary to carry out the tasks of teaching mathematics as a way to access their concepts and beliefs about the work of teaching mathematics. Furthermore, it is argued that any understanding of what the work of teaching (from the perspective of pre-service teachers) entails will be severely limited unless it incorporates an understanding of how pre-service teachers make sense of what teachers must know and do in the classroom (Brown & McIntyre, 1993). These constructs are central to educational research and are not usually core-content areas in teacher education programs, especially in countries where the educational system is under development.

A first step in developing this study was to formulate the research question and the possibilities to contribute to the field of teacher education. In Chapter 1, I explained that this process was initially based on on-site visits to primary schools and teacher education colleges in Malawi and reviews of literature and research conducted on the topic of teacher education and teacher knowledge. Moreover, by assuming that education is a crucial element to the successful implementation of the 2030 agenda for sustainable development by UNESCO (2019), I argued that efforts targeting the improvement of teacher education should consider forms of not only improving access to education but also the quality and conditions for pre-service teachers to develop proficiency in teaching.

In Malawi, the redesign of the IPTE two-year program (Chapter 3) led to a set of principles that emphasize the need for pre-service teachers to develop an in-depth understanding of mathematical knowledge for teaching in primary schools (Malawian Institute of Education, 2017). Its syllabus encompasses standards that require pre-service teachers to acquire subject matter knowledge and develop pedagogical ways to promote students' lifelong learning (Malawian Institute of Education, 2017). Although such a proposal seemed to be promising, the views that sustain the document were based on what the pre-service teachers should learn rather than what they already know or are capable of doing, especially at the beginning of their teacher education. Findings from Jakobsen et al.'s (2018) indicate possible gaps in this curriculum proposal as pre-service teachers still showed low level of improvements in their mathematical knowledge for teaching at the end of the program. Thus, there is a clear need for more awareness on what pre-service teachers know and have learned in their teacher education rather than what should be taught to them.

In this thesis, my goal was to provide a comprehensive picture of the pre-service teachers' understandings of the tasks and knowledge necessary for teaching mathematics and how they develop such understandings throughout their teacher education. In this thesis, I argued that it is important to consider the needs and specificities of pre-service teachers regarding the knowledge entailed in the work of teaching. This thesis, therefore, marks a major step in the field by providing new data and insights on this a topic.

7.1 Responding to the Research Question

The overall research question of this study was: *How do pre-service teachers develop their understanding of the knowledge necessary* *to teach mathematics throughout teacher education?* This question was divided into three specific research questions that intended to cover the general question (Chapter 1).

In response to the first sub-research question, *What understanding do pre-service teachers have of the knowledge needed to carry out the tasks of mathematics teaching at the beginning of their teacher education?*, I intended to get a better sense of the types of knowledge that pre-service teachers in Malawi understand to be crucial to carry out the tasks of teaching mathematics. I found that entering primary pre-service teachers in Malawi acknowledged the relevance of most types of knowledge widely accepted by the literature (Ball et al., 2008). Despite some variabilities, they tended to justify and make sense of teaching knowledge as being more closely related to practical experiences and the instructional guidelines from the IPTE program and school curricula.

Additionally, at the beginning of the teacher education program, Malawian pre-service teachers tended to view mathematics teaching as being dependent on the knowledge of content and the ability to use locally available resources in mathematics instruction. When the pre-service teachers were asked about the knowledge that is not part of the instructional curriculum, few understood it as relevant to teaching mathematics. For them, this broader type of knowledge is crucial to facilitating the work of teaching in meeting the curriculum demands and helping students to get a general idea of the origins of mathematical content, but it was not as important for helping students to solve practical mathematical problems in Malawi.

Nonetheless, issues like overcrowded classrooms and lack of didactical resources can diminish the significance of teachers' SCK in carrying out tasks of teaching mathematics, as indicated by the pre-service teachers at the beginning of teacher education. They were broadly consistent in explaining the specificities and connections between the tasks of teaching mathematics and the knowledge that are unique to the work of teaching, as described by Ball et al. (2008). Thus, as suggested in the present study, entering pre-service teachers in Malawi hold a partial and intuitive understanding of the knowledge needed to carry out the tasks of teaching mathematics in primary schools. This initial understanding was crucial to comprehend the pre-service teachers' learning process throughout teacher education.

In addressing the second sub-research question, *To what extent does the pre-service teachers' understanding of the knowledge needed to carry out tasks of teaching evolve through the discussion of practical experiences in college?*, I aimed to examine the pre-service teachers' understanding of the tasks of teaching and knowledge demands that manifested in their own mathematics lessons during teaching practice in local schools. I found that, at the stage of teaching practice, the pre-service teachers could better describe the tasks of teaching that were necessary to fulfill the work of teaching mathematics. I also noticed that after teaching practice, pre-service teachers followed unique paths of evolution regarding their understanding of the knowledge needed to carry out these tasks effectively. At this stage, most of them became more aware of the limits in only considering the instructional guidelines proposed by the IPTE curriculum and school textbooks. The pre-service teachers recognized the need to develop broad knowledge and skills to create more consistent activities to meet students' needs and specificities of learning. As a result, pre-service teachers appeared to expand their intuitive understanding from the beginning of the program to a more complex one during teaching practice, even though they still endorse the institutional curricular guidelines.

In the third sub-research question, *How do pre-service teachers develop their understanding of the knowledge needed to carry out tasks of teaching throughout teacher training?*, I sought to analyze the dialogue, arguments, and reflection that pre-service teachers made during a focus-group discussion at the end of teacher education. To complement such an analytical process, I combined data from the two aforementioned moments of the research. Thus, by taking a development perspective of the understanding that pre-service teachers revealed of teaching knowledge over the course of three research moments, I found that throughout teacher education, pre-service teachers developed a more nuanced understanding of the knowledge needed to carry out tasks of teaching mathematics in primary schools. They could clearly articulate their thoughts and ideas with principles suggested by the theories emphasized in the IPTE curriculum (Malawian Institute of Education, 2017).

By the end of teacher education, in particular, the pre-service teachers tended to reframe their initial understanding of teaching knowledge to a more useful form of application. For instance, in Article 4, Mario's reflections on tasks of teaching that are unique to the work of primary school teachers in Malawi showed he was aware of possible connections between different types of knowledge to carry out this task; he made several references to the knowledge of how to use locally available resources to create multiple representations to teach mathematics. Also, as he explained the difference of this knowledge with his initial understanding presented at the beginning of the program, it was clear that he had critical thought on the limits and potentialities of SCK to teach mathematics, showing that his intuitive understanding of teaching knowledge had prolonged to be more sophisticated.

The conclusions above demonstrate that entering Malawian pre-service teachers tend to develop their understanding of the knowledge necessary to teach mathematics as they progress through teacher education. Such a development process, however, is complex and closely dependent on the activities and experiences they have during teacher education. As pre-service teachers gain more experience throughout studies about theories in the coursework, teaching classes in local schools, and reflection activities about their learning afterwards, it can be assumed they can revise and improve their intuitive ideas and thoughts in the light of their future role as teachers as well as of the theoretical constructions taught by the teacher education program. Therefore, the understanding pre-service teachers develop of the knowledge needed to carry out the tasks of teaching mathematics during teacher education might constrain the opportunities for conceptual development of essential theoretical constructs that are unique to the work of teaching if they are given the opportunity to think, experience and reflect. However, it was unclear if such awareness could indicate whether pre-service teachers' understanding of teaching knowledge can continue to improve after teacher education.

7.2 Limitations of the Study

Examining the understanding that pre-service teachers develop of the knowledge necessary to teach mathematics throughout teacher education imposes challenges for any study, particularly for a qualitative study.

One weakness of this study was the lack of a theoretical foundation based on previous research on the topic. Although the current study can be situated in the context of research about teachers' epistemic belief of teaching knowledge, there was very little, or no similar research that focused on the understanding pre-service teachers develop of the knowledge necessary to carry out tasks of teaching mathematics during teacher education (Chapter 2). Similar research on this topic had explored the teachers' mathematical knowledge for teaching as a theoretical construct, teachers' ability to articulate theoretical constructs of mathematical knowledge for teaching and practice, and teachers' knowledge of the tasks of teaching and skills demands to carry out these tasks in mathematics (see Chapter 2). Thus, the scope of works that have been done in consonance with the current study is limited and provides only limited support to understand the research problem investigated.

The case study sample size also limited further conclusions of this study. Many of the twenty-three pre-service teachers who participated in the IM of the study provided relevant data. As the analyses focused only on six cases of pre-service teachers, a significant set of data could not be considered for this thesis. A larger sample—perhaps collected in different teacher education colleges—may have provided different insights into the focus of this study.

A third limitation of this study relates to conditions given to conduct the teaching observations in local schools. As explained in Chapter 4, after taking coursework during the two first terms at the college, pre-service teachers in Malawi went to local schools for practical teaching experience. Some schools were located in remote regions across the country, mostly in rural areas, making it difficult for the researcher to access those locations. Even with the help of some colleagues and teacher educators from the teacher college and the University of Malawi, it was difficult to trace the schools and have appropriate time to conduct the interviews with the pre-service teachers.

After accomplishing all the three moments of the study, I observed that it could have been done differently. If I had more time in my Ph.D. program, it probably would have been easier to have all the data from three moments at once, and transcribe and analyze the data later rather having to collect data from each moment, and transcribe and analyze before the subsequent moments. The analysis would have probably been more consistent and coherent with the focus of the study. Also, the study could have included new forms of analysis by analyzing pre-service teachers' cases from other teacher education programs. This could have been an encouraging way to find out whether the pre-service teachers' views and understanding of teaching knowledge correspond, an aspect that the current study did not cover.

7.3 Implications for Practice

One practical contribution of this study relates to teacher educators' teaching activities. Overall, the results showed that pre-service teachers revealed different forms of understanding of the knowledge necessary for teaching mathematics throughout teacher education, a condition that suggests that pre-service teachers need distinct types of support and guidance to convene their learning needs during teacher education.

This study helps teacher educators to develop a better sense of how pre-service teachers understand the tasks and knowledge demand for teaching mathematics. Teacher educators could use the results of this work to improve their teaching to determine the most appropriate tasks and activities that could stimulate pre-service teachers to expand their views and knowledge about mathematical knowledge for teaching.

Teacher educators could implement the cases explored in this study as a teaching strategy to engage their students with reflections on how the participants of this study carried out the tasks of teaching during practical situations. Using case studies to prepare pre-service teachers for teaching will likely be an effective way for teacher educators to improve their teaching as the instances used are research-based cases. Pre-service teachers will have more opportunities to discuss and reflect on the lived experience and views that others had in their teacher education or professional development. The idea is not to make pre-service teachers think or employ the same strategy from other cases, but instead, help them become more familiar with the challenges of the work of teaching so they can use their academic skills to carry out tasks of teaching in practical contexts.

This thesis also has implications for future curricular reforms of teacher education guidelines in Malawi. As shown in Article 1, pre-service teachers acknowledge the relevance of mathematical knowledge beyond the primary school curriculum. However, the syllabus for teacher education in mathematics in Malawi (Malawian Institute of Education, 2017) does not yet propose further discussion or activities for pre-service teachers to develop such knowledge. Another type of knowledge that is not considered in detail in this document is the specialized content knowledge, a unique and crucial component for the education of teachers in mathematics (Ball et al., 2008). As reported in Articles 3 and 4, although aligned with Ball et al.'s (2008) definition of specialized content knowledge, pre-service teachers still seem to make more complex interpretations of this knowledge when reflecting practical situations. Future curricular reforms could consider implementing learning activities that encourage pre-service teachers to develop a better understanding of this knowledge so they can be better prepared to carry out mathematics teaching tasks. The curriculum could also include or describe how its principles and philosophy relate to UNESCO's (2015b) Sustainable Development Goals.

7.4 Methodological Implications

The overall study methodology was drawing upon the use of multiple data collection instruments in different phases of the fieldwork. In educational research, it is common to find case studies that explore a case using similar instruments and procedures repeatedly over time to maintain the reliability of the results, but as Parlett and Dearden (1977) reminded, a case study should also be flexible in terms of design as "the course of the study cannot be charted in advance" (p. 15). While this approach increases the complexity and amount of data that are evaluated, the use of several tools and procedures increases the reliability and validity of the study by avoiding problems and researchers' bias and levels of analysis, supports Stake (2005). Thus, the methodology of the present study serves as a model for longitudinal case studies that aim to collect data from many sources and shed light on the research question from many angles. This is particularly relevant for qualitative research where one phase of the study contributes to the elaboration and implementation of the next phases of the study, providing steady insights.

Beyond replicability for future research, a final methodological contribution of this thesis is the transparency of the methods used throughout the study. Transparency refers to the explicit information of the objectives, methods, and results of a study that has been implemented, and to how the study was interpreted and presented by the researcher(s) (Creswell, 2005). Sufficient transparency in qualitative research is of particular importance for clarifying the research settings, sampling procedures, and unexpected situations or challenges faced by the researcher (Aguinis & Solarino, 2019). The current study sought to provide explicit details and transparency around elaboration, development, and outcomes, so the presentation and final publications of this thesis can be used as an example for future research on the topic.

7.5 Theoretical Implications

A theoretical contribution of this thesis relates to the study of the tasks and knowledge needed for teaching. Researchers have drawn larger and smaller knowledge essential for teaching mathematics based on tasks identified during practical teaching of mathematics (e. g., Mitchel et al., 2014; Ball, 2017). This thesis examined the understanding pre-service teachers develop of the tasks of teaching and knowledge needed to teach mathematics in the context of Malawi. The empirical work shows that such a context reveals additional forms of tasks and knowledge demands for teaching mathematics (e.g., knowledge of how to select and convert locally available resources into learning materials in mathematics). The study of distinctive forms of the tasks and knowledge needed to carry out these tasks increases not only the understanding of the teaching experience and practice across different contexts but also contributes to the refinement or development of a framework for analyzing and understanding teaching, much needed in teacher education (Grossman & McDonald, 2008). Nonetheless, researchers can benefit from a greater understanding of how this knowledge might differ as context changes are necessary (Kazima et al., 2016).

Another theoretical contribution of this thesis can be related to the conceptual framework used in this study. In recent years, a few investigations have emerged in the field of teacher education concerning beliefs of teaching knowledge. Among the most promising works are those of Buehl and Fives (2009) and Mosvold and Fauskanger (2013), who provided theoretical tools to assess teachers' epistemic beliefs of teaching knowledge in mathematics. Although these are significant works for examining pre- and in-service teachers' beliefs of teaching knowledge, the lack of a clear definition of epistemic belief remains a concern in the field since scholars might denote it as knowledge about the nature of knowledge and knowing (Hofer & Pintrich, 1997) or as a lens through which a person interprets a phenomenon or situation (Rebmann et al., 2015).

In this thesis, while increasing engagement to conduct the empirical and analytical work was based on the researcher's personal experiences and educational principles (see Chapter 1), the outcome of this thesis might also contribute to the better clarification of the realm in which teachers' understanding of the tasks and knowledge necessary to teach mathematics can best be investigated in educational research. A combination of Smith and Siegel's (2004) criteria for understanding and Ball et al.'s (2008) domains of mathematical knowledge for teaching can be a more efficient way to structure and guide empirical and analytical studies about teachers' understanding of the knowledge necessary to teach mathematics. Although, the current study might shed light on the understanding of what epistemic belief is—defined by Philipp (1997) as "psychologically held understandings, premises, or propositions about the world that are thought to be true", it is an example that the development of a cognitive aspect of pre-service teachers' understanding can be studied and used to improve researchers' understanding of the nature and mathematical knowledge for teaching from the perspective of who are preparing to become teachers in mathematics.

7.6 Further research

The current research shows that pre-service teachers can develop their understanding of teaching knowledge as they progress in teacher education. The study sought to obtain a better picture of the types of tasks and knowledge demands that pre-service teachers though to be most crucial for teaching mathematics. The study also took a development perspective of this understanding in different moments of the teacher education program and concluded that pre-service teachers develop their understanding in different paths closely related to the curricular principles taught during teacher education. This conclusion, however, raises other questions, such as: Why did the pre-service teachers have different paths of development for their understanding of teaching knowledge? Would their understanding be the same if they had to teach the same mathematical topic? What other engaging teaching methods could create opportunities for pre-service teachers to develop their understanding of teaching knowledge? What kind of results will come out if pre-service teachers examine tasks of teaching mathematical lessons from teachers in different countries?

In relation to the last question, future research could consider what is happening across the globe by offering examples and insights of how pre-service teachers have developed their views and understanding of teaching knowledge during teacher education. These insights could be used to develop research beyond case studies such as cross-sectional research or action research. Findings from these other types of research would be important to the field, as it could increase our understanding of what we already know about teacher education and teacher knowledge.

7.7 Personal Growth

7.7.1 As a Researcher

I started the journey of this research enthused about how pre-service teachers understand the knowledge necessary to teach mathematics, hoping that what would be learned could lead to a better understanding of how they develop mathematical knowledge for teaching and use it to create teaching activities that could help students to develop their best qualities as a human beings. However, throughout this research, I also developed some personal attributes and discovered several new ideas about teacher education.

I was always fascinated by the work of teaching. I was mostly intrigued by some teaching activities that could awaken students' curiosity and attention and make them work together to solve a problem. I always asked myself: Is that an activity that I could use to make my students better thinkers? If not, what could I do to make it more useful and meaningful? Most of the time, these questions led me to think about the quality of knowledge and skills necessary to create these activities.

During my Ph.D. research, I read and studied an abundance of educational literature about the knowledge teachers need to create effective teaching activities in the mathematical classroom. Above all the theories of teaching knowledge, I found Ball et al.'s (2008) ideas to be the most remarkable as they tried to map the tasks and knowledge that teachers must execute to teach mathematics effectively. Such an attempt did not only call the scientific community's attention but also opened a greater avenue for researchers to explore, refine, and adapt these components in different contexts of teaching. Their theory inspired me to explore the complexity of the work of teaching and persevere on the journey to improve the quality of education. I recognized my professional growth as a researcher, along with the theoretical principles of the mathematical knowledge for teaching realms.

Furthermore, designing the research was an exciting task for me. The whole process has been an entirely new experience, as I was challenged by employing a possible methodology that would be consistent enough for interpreting the understanding of pre-service teachers who developed the knowledge necessary to teach mathematics during teacher education. I learned that having a well-structured methodological plan is not a guarantee that the researcher will be able to collect a good set of data. It is also necessary that the research take some risks (emotionally and physically) by interacting with the community, live and experience their agonies, and develop a spirit of improvisation and optimism.

7.7.2 As a Teacher Educator

As a teacher educator, it has been an illuminating opportunity to see how pre-service teachers learn, experience, and reflect on key aspects of their future profession. When I observed the pre-service teachers teaching their lessons to the children, I was reminded the importance of a teacher to a student and how much a children can be grateful when just a little attention is given to them. I always thought that planning a logical, well-organized, and high-tech activity could make students more engaged in learning mathematics. Still, after this research, I see that having a well-equipped classroom or an answer to every single question you make is not actually the primary purpose of education. The main goal is to create opportunities for students to develop their curiosity and critical thinking so they can learn how to learn and become lifelong learners. This was something that would not be possible without conducting this research with the pre-service teachers in Malawi. I was amazed at how pre-service teachers acquired and constructed the idea of effective teaching and kept working to achieve it regardless of the obstacles they faced in school. It was really a lesson as a teacher educator who seeks to encourage teachers worldwide to explore new forms of teaching mathematics that help students become better persons.

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Appendices

Appendix 1 – Questionnaire Survey

This questionnaire is an integral part of a research proposal that investigates the pre-service teacher´ sense-making process about mathematical knowledge for teaching (MKT)[3]. It is designed to assess, upon the beginning of the teacher-training program, the perceptions of pre-service teachers about MKT in primary schools in Malawi. The questions below do not have a right or wrong answer; they seek to collect personal information and opinions. The questionnaire is confidential, and its answer will be used exclusively for research purposes. The questionnaire is divided into two parts: Identification and Mathematical knowledge for teaching.

Identification

a) Name:

b) Age: ❑ Less than 17 years old; ❑ From 18 to 20 years old; ❑ From 21 to 23 years old; ❑ From 24 to 27 years old; ❑ From 28 to 30 years old; ❑ More than 31 years old.

c) What language(s) do you speak? *(Native language, dialectics, second language…)*

d) What was (were) your favorite subject(s) in school?

[3] *Research project grounded by the University of Stavanger, supervised by Prof. Arne Jakobsen, Prof. Raymond Bjuland, Prof. Tone Bulien, and Prof. Mercy Kazima (University of Malawi).*

❑ Geography ❑ Biology ❑ Arts ❑ Mathematics
❑ Other:

Why?

e) Do you have any teaching experience prior to entering the teaching training college? If yes, could you describe this experience?

f) What are your main subjects in the teacher-teaching program? *(Arts, Biology, Geography, Mathematics…).*

g) Have you taken any course in mathematics or mathematics teaching prior to entering the teaching-training program? If yes, please describe it.

Relevance of the Mathematical knowledge for Teaching

In your opinion, to what extent are the knowledge and skills below important for the work of teaching mathematics in primary school?

Knowledge and skills necessary to teach Mathematics	Not al all important	Slightly important	Moderately important	Very important	Extremely important
a) Knowledge of mathematical concepts contained in the school curriculum					
b) Knowledge of mathematical concepts outside (beyond those prescribed by) the school curriculum					
c) Using mathematical terms and notations correctly					
d) Ability to verify the accuracy of the mathematical definitions and examples in the textbooks					
e) Ability to identify student errors					
f) Ability to identify and explain where and how the students' errors occur					
g) Understanding the methods proposed by the textbooks to solve mathematical problems					
h) Knowledge of the different methods (generally not contained in the textbooks) that can be adopted to solve mathematical problems					
i) Ability to introduce mathematical concepts in exactly the same way as the textbook suggests					

j) Knowing how to introduce mathematical concepts and procedures in different ways (usually not contained in the textbooks)					
k) Ability to prove mathematical statements					
l) Knowing how to give contextual examples for underlying mathematical ideas					
m) Knowledge of advanced mathematics					
n) Ability to engage students in the learning process					
o) Ability to explain how a particular mathematical topic is related to another mathematical topic					
p) Ability to explain how a mathematical topic is associated with other subject(s)					
q) Knowing how to link mathematical topics to real-life situations					
r) Ability to organize activities outside the classroom					
s) Anticipating students' errors and common misconceptions					
t) Interpreting students' ideas correctly					
u) Ability to predict what students are likely to do when given specific tasks and what they will find interesting or challenging					
v) Knowing how to design a lesson effectively					

w) Ability to connect a lesson with another lesson					
x) Ability to choose appropriate exercises for instructional tasks					
y) Knowing how to use different teaching methods to teach mathematics					
z) Ability to evaluate the advantages and disadvantages of using specific representations					
aa) Comprehending the purposes and ethical values of education					
bb) Knowledge of the history of mathematics					
cc) Knowing how different cultures (or nations) develop and use mathematical ideas					
dd) Willingness to conduct research to uncover new facts and ideas related to the curriculum					
ee) Understanding what students find interesting and motivating in mathematics					
ff) Knowing what students find easy and difficult in learning mathematics					

Appendix 2 – Interview Guide

Complementary Interview guide used by the researcher

This interview guide is compost of eight open-ended questions related to the first moment of research that investigates the pre-service teacher´s understanding (and its evolution process throughout the theoretical training) of mathematical knowledge for teaching in primary schools. These questions do not pretend to obtain definitive answers but aim to discuss the motivations, necessities, and the ways in which the participants acknowledge the activities and experiences developed during their teacher education. The estimated time to cover all six questions is 50 minutes.

1) Why did you apply for this teacher-training program?

2) Could you walk me through what you have done in the first term of the program? *(Disciplines, mathematical contents, study activities…)*

3) Could you describe a typical day in the discipline of teaching mathematics?

4) Could you give me an example of a time in classroom that you enjoyed and disliked? Why?

5) How is the mathematical content you are learning so far important for teaching mathematics in primary school?

6) (This question considers the answers given throughout the questionnaire). To what degree do you think you possess the knowledge that you marked as mostly important for teaching mathematics in primary schools?

7) Since you started the teacher-training program have you become more or less motivated in becoming a teacher?

8) Do you feel that teacher-training program is preparing you to teach mathematics for primary school children?

Appendix 3 – NSD Approval

NSD

Everton Lacerda Jacinto

4036 STAVANGER

Vår dato: 12.04.2018	Vår ref: 60052 / 3 / OASR	Deres dato:	Deres ref:

Vurdering fra NSD Personvernombudet for forskning § 31

Personvernombudet for forskning viser til meldeskjema mottatt 23.03.2018 for prosjektet:

60052	*Preservice teachers´ sense-making process of the work of teaching in primary schools in Malawi*
Behandlingsansvarlig	*Universitetet i Stavanger, ved institusjonens øverste leder*
Daglig ansvarlig	*Everton Lacerda Jacinto*

Vurdering

Etter gjennomgang av opplysningene i meldeskjemaet og øvrig dokumentasjon finner vi at prosjektet er meldepliktig og at personopplysningene som blir samlet inn i dette prosjektet er regulert av personopplysningsloven § 31. På den neste siden er vår vurdering av prosjektopplegget slik det er meldt til oss. Du kan nå gå i gang med å behandle personopplysninger.

Vilkår for vår anbefaling

Vår anbefaling forutsetter at du gjennomfører prosjektet i tråd med:
- opplysningene gitt i meldeskjemaet og øvrig dokumentasjon
- vår prosjektvurdering, se side 2
- eventuell korrespondanse med oss

Vi forutsetter at du ikke innhenter sensitive personopplysninger.

Meld fra hvis du gjør vesentlige endringer i prosjektet

Dersom prosjektet endrer seg, kan det være nødvendig å sende inn endringsmelding. På våre nettsider finner du svar på hvilke endringer du må melde, samt endringsskjema.

Opplysninger om prosjektet blir lagt ut på våre nettsider og i Meldingsarkivet

Vi har lagt ut opplysninger om prosjektet på nettsidene våre. Alle våre institusjoner har også tilgang til egne prosjekter i Meldingsarkivet.

Vi tar kontakt om status for behandling av personopplysninger ved prosjektslutt

Ved prosjektslutt 30.11.2020 vil vi ta kontakt for å avklare status for behandlingen av

Dokumentet er elektronisk produsert og godkjent ved NSDs rutiner for elektronisk godkjenning.

NSD – Norsk senter for forskningsdata AS   Harald Hårfagres gate 29   Tel: +47-55 58 21 17   nsd@nsd.no   Org.nr. 985 321 884
NSD – Norwegian Centre for Research Data   NO-5007 Bergen, NORWAY   Faks: +47-55 58 96 50   www.nsd.no

personopplysninger.

Se våre nettsider eller ta kontakt dersom du har spørsmål. Vi ønsker lykke til med prosjektet!

Marianne Høgetveit Myhren

Øivind Armando Reinertsen

Kontaktperson: Øivind Armando Reinertsen tlf: 55 58 33 48 / Oivind.Reinertsen@nsd.no

Vedlegg: Prosjektvurdering

NSD

NOTIFICATION FORM

Notification form (version 1.5) for student and research projects subject to notification or license
(cf. the Personal Data Act, the Personal Health Data Filing System Act and associated Regulations).

1. Intro		
Will directly identifiable personal data be collected?	Yes ● No ○	A person will be directly identifiable through name, social security number, or other uniquely personal characteristics.
If yes, please specify	■ Name □ Social security number □ Address □ E-mail □ Phone number ■ Other	Read more about personal data. NB! Even though information is to be anonymised in the final thesis/report, check the box if identifying personal data is to be collected/recorded in connection with the project. Read more about what processing personal data entails.
If other, please specify	Year of Birth and Gender.	
Will directly identifiable personal data be linked to the data (e.g. through a scrambling key/reference number which refers to a separate list of names)?	Yes ● No ○	Note that the project will be subject to notification even if you cannot access the scrambling key, as the procedure often is when using a data processor.
Will there be collected background information that may identify individuals (indirectly identifiable personal data)?	Yes ○ No ●	A person will be indirectly identifiable if it is possible to identify a person through a combination of background information (such as place of residence or workplace/school, combined with information such as age, gender, occupation, etc.).
If yes, please specify		NB! In order for a voice to be considered as identifiable, it must be registered in combination with other background information, in such a way that a person can be recognized.
Will there be registered personal data (directly/indirectly/via IP or email address, etc.) using online surveys?	Yes ○ No ●	Read more about online surveys.
Will there be registered personal data using digital photo or video files?	Yes ● No ○	Photo/video recordings of faces will be regarded as identifiable personal data.
Have you applied for an assessment from REC regarding whether the project should be considered health research?	Yes ○ No ●	NB! If REC (Regional Committees for Medical and Health Research Ethics) has assessed the project as health research, you do not have to submit a notification form to the Data Protection Official. (NB! This does not apply to projects using data from pseudonymous health registers.) Read more. If you have not received a reply from REC, we recommend that you await filing out a notification form until you have received a reply.
2. Project Title		
Project Title	Preservice teachers' sense-making process of the work of teaching in primary schools in Malawi	Please state the project title NB! This cannot be "Master's thesis" or the like, the must describe the content or aim of the project.
3. Responsible Institution		
Institution	Universitetet i Stavanger	Select the institution to which you are affiliated. All administrative levels must be specified. If it is a student project,select the institution to which the student is affiliated. If your institution is not listed, please contact the institution. Read more about the responsible institution. (data controller).
Section/Faculty	Fakultet for utdanningsvitenskap og humaniora	
Department	Institutt for grunnskolelærerutdanning, idrett og spesialpedagogikk	
4. Project Leader (Researcher, Supervisor, Research Fellow)		

Page 1

Personvernombudet for forskning

Prosjektvurdering - Kommentar

Prosjektnr: 60052

According to your notification form the sample will receive written and oral information and will give their consent to participate. The information letter we have received is well formulated. However, we ask that the following is added:
- Give the date for the end of the project and information about what will happen to the personal data (you have notified that the data will be anonymized).

The Data Protection Official presupposes that research is conducted in line with laws, regulations and guidelines in Malawi.

The Data Protection Official presupposes that you will process all data according to the Universitetet i Stavanger internal guidelines/routines for information security. We presuppose that the use of a mobile storage device is in accordance with these guidelines.

The estimated end date of the project is 30.11.2020. According to your notification form/information letter you intend to anonymise the collected data by this date. Making the data anonymous entails processing it in such a way that no individuals can be identified. This is done by:
- deleting all direct personal data (such as names/lists of reference numbers)
- deleting/rewriting indirectly identifiable personal data (i.e. an identifying combination of background variables, such as residence/work place, age and gender)
- deleting video files.

First name	Everton	Fill in the name of the person who will have the day-to-day responsibility for the project. In a studentproject, this will usually be the student's supervisor. Read more about the role of the project leader.
Surname	Lacerda Jacinto	
Position	PhD Research Fellow	The student and the supervisor should usually be affiliated with the same institution. If the student has an external supervisor, the assistant supervisor at the student's place of study should be registered as the project leader.
Telephone	+47	
Mobile	92536903	
Email	everton.l.jacinto@uis.no	Place of work must be affiliated with the responsible institution, e.g. a department, institute or section.
Alternative email	lacerdajacinto@gmail.com	Please notify us if you change your e-mail address.
Place of work	University of Stavanger	
Address (work)	Faculty of Arts and Education	
Postcode/city (work)	403 STAVANGER	
5. Student (master, bachelor)		
Student project	Yes ○ No ●	If the project will be carried out by more than one student, please choose one as contact person. Remaining students can be added under question 10.
6. Objective		
What is the purpose of the project?	The purpose of the project is to understand how preservice teachers in Malawi make sense of the mathematical knowledge for teaching (MKT) in primary schools. The three main research questions are: To what extent do preservice teachers´ perceptions about MKT evolve throughout the theoretical disciplines of teacher training? How can reflective practice influence the evolution of preservice teachers perceptions about MKT? How do preservice teachers negotiate their (new) perceptions about MKT in practical applications?	Briefly describe the purpose or theme of the project and/or the research question.
7. Sample		
Please specify your sample	□ Children attending day-care institutions □ School children □ Patients □ Users/clients/customers □ Employees □ Children connected to child welfare ■ Teachers □ Health/medical personnel □ Asylum seekers ■ Other	Read more about different research topics and samples.
Describe the sample/participants	Preservice teachers in one of the eight teacher training college for primary schools in Malawi. 1 or 2 Mathematics teacher educators at the teacher training college will also be interviewed.	The sample refers to those who participate in the study or whom you collect information about.
Recruitment and sampling	One of the eight teacher training colleges in Malawi is purposely selected (due to geographical location). At this college, a questionnaire will be applied in two different classes composed of 25 preservice teachers each. Based on their answers, 3 to 4 groups of 2-3 preservice teachers will be chosen for interviews and observations regarding their learning process and teaching activities throughout the teacher training.	Briefly describe how the sample will be recruited/sampled. The sample may be recruited through a company, a school, a sports organisation, your social network, etc, or drawn from registries, such as the National Registry or patient registries. Please specify who will recruit/draw the sample.
Initial contact	The initial contact is through the principal at the teacher training college who will receive information about the project and will grant access to the teacher training college. The principal will introduce me to the mathematics teachers. I will introduce the project to the students (the prospective teachers) and I will ask students to answer a questionnaire and I will underline that this is voulentairely to do. They who sign the consent form will be part of the project. Based on answer on questionaire, groups of students for interview will be selected.	Describe how initial contact withe the sample will be established, and by whom. Read more about initial contact and different samples on our pages on: research topics.

Page 2

Sample age	□ Children (0-15 years old) □ Adolescents (16-17 years old) ■ Adults (18 years and/or older)	Read about research involving children on our web pages.
Approximate number of sample/participants	25 for the first phase and 6 to 8 for the second phase	
Will sensitive personal data be collected?	Yes □ No •	Read more about sensitive personal data.
If yes, please specify	□ Racial or ethnic origin, or political opinions, philosophical or religious beliefs □ The fact that a person has been suspected of, charged with, indicted for or convicted of a criminal act □ Health □ Sex life □ Trade-union membership	
Will legal adults with impaired consent capacity be included in the sample?	Yes □ No •	Read more about the inclusion of persons with impaired consent capacity.
Will there be collected personal data about persons who do not directly participate in the study? (third persons)	Yes □ No •	Information about third persons is information which can (directly or indirectly) identify individuals not included in the sample, e.g. a colleague, student, client, family member. Read more.
8. Method(s) for Data Collection		
Please specify method(s) for data collection	■ Paper-based questionnaire □ Electronic questionnaire ■ Personal interview ■ Group interview ■ Observation ■ Participant observation □ Blog/social media/internet □ Psychological/pedagogical tests □ Medical exams/tests □ Records (medical records)	Personal data can be obtained directly from the data subject e.g. from a questionnaire, a personal interview, tests, and/or various records (medical records, nursing home, the Labour and Welfare Administration (NAV), etc.) and/or from existing registries (e.g. Statistics Norway). NB! If personal data will be collected from diverse persons (samples) or using diverse methods, this must be specified in the Comment-box. Remember to enclose relevant attachments for all the different groups/samples and methods to be used. Read more about register-based studies. If you will be using data from registries, a list of variables must be uploaded at the end of the notification form. Read more about research methods.
	□ Registries	
	□ Other	
Comment	The methods for data collection will be applied in four distinct moments. The first two moments make only use of questionnaire and interviews. The other two moments combine interviews, observations and discussions about the practical activities recorded.	
9. Information and Consent		
Specify how the sample will be informed about the project	■ Written ■ Verbal □ They will not be informed	If the sample will not be informed about the processing of personal data, you must state the reason for this. Read more Please attach a copy of the information letter/script for verbal information. Download an example of our Information Letter. Read more about requirements for consent. NB! Attachments can be uploaded at the end of the form.
Will the sample be asked to give their consent?	• Yes ○ No ○ Several samples, not consent from all	In order to define a consent to participation in research as valid, the consent must be freely given, specific and informed. A consent may be given in writing, orally or by action. For instance, filling out a questionnaire will be regarded as an active consent. If consent is not to be obtained, you must state a reason. Read more.
10. Information security		

Page 3

How/where will the list of names be stored, and who will have access to it?	The participant's names will be codified into Preservice teachers PSTA, PSTB, PSTC, PSTD, PSTE, PSTF, PSTG, PSTH, and the preservice mathematics teacher educator(s) into PSMTA and PSMTB. The data will be accessed by the project leader.	
Will directly identifiable information be registered/stored in other ways?	Yes ● No ○	
Please specify	Video of interviews will be recorded and stored safely and secured through the project time	NB! As a general rule directly identifiable personal data should not be registered together with the rest of the data. We recommend the use of a scrambling key
How will personal data be registered and stored/processed?	□ On a server in a network belonging to the institution ■ An isolated computer belonging to the institution (i.e. with no connection to other computers or networks, internally or externally) □ A computer in a network with internet access belonging to the institution □ Private computer ■ Video recordings/photographs □ Audio recordings □ Manually/on paper ■ Portable storage device (laptop, USB , memory card, CD, external hard drive, mobile phone etc.) □ Other	Please specify each of the different ways the data will be registered/processed. You may check more than one box if applicable. By "institution" we mean the institution responsible for the project. NB! As a general rule, personal data should be stored on a research server belonging to the responsible institution. Using other media for storing - such as private computer, mobile phone, USB, external server - is less secure, and must therefore be given account for. Such storing must also be clarified with the responsible institution, and personal data should be encrypted.
Other, please specify		
How will the data be protected from unauthorized access?	After video recordings, videos will be copied to a portable storage device. This device will have a password (eight characters longs with small and capital letters and numbers) and the device will be stored in locked office desk in a locked office. During transcription of data, part of the data will be on an isolated computer belonging to the institution.	For instance, will the computer be password protected, will the computer be kept in a locked room, how will portable units, printouts, recordings etc. be protected from unauthorized access?
Will data be gathered/processed by an external processor?	Yes ○ No ●	External processor refers to someone who gathers or in other ways processes personal data on behalf of the person/institution responsible for the project e.g. supplier of electronic questionship, transcription service provider or interpreter. These assignments must be regulated by a contract. Read more
If yes, please specify		
Will personal data be gathered or transferred through e-mail/the internet?	Yes ○ No ●	For instance by transferring data to a collaborator, data processor etc. If personal data is to be sent by email, the information should be adequately encrypted. We do not recommend storing personal data on cloud services. Using cloud services must be clarified with the responsible institution. As a general rule, if cloud services are used, a data processor agreement with the supplier of the service must be signed. Read more.
If yes, please specify		
Will there be others working on the project, in addition to the project leader/student, who will have access to personal data?	Yes ● No ○	
If yes, who? (name and place of work)	My PhD supervisors: Professor Arne Jakobsen (University of Stavanger), Professor Raymond Bjuland (University of Stavanger), Associate professor Tone Bulien (University of Stavanger) and Professor Mercy Kazima (University of Malawi).	
Will personal data be shared with national or international institutions?	● No ○ Yes, national institutions ○ Yes, institutions in other countries	I.e. in national or international multicenter studies where personal data is shared.
11. Assessment/approval by other regulating bodies		
Will your project require a dispensation from the duty of confidentiality in order to gain access to data?	Yes ○ No ●	In order to gain access to information which is subject to duty of confidentiality, e.g. data from hospitals, the Labour and Welfare Administration (NAV), or other public institutions, you must apply for a dispensation from the duty of confidentiality. A dispensation is normally granted by the relevant government department.
If yes, please specify		

Page 4

Will your project require assessments/approval from other regulating bodies?	Yes ● No ○	In some research projects, it may be necessary to seek additional permissions. For instance, the registry owner may need to grant access to data, or permission may be needed from the management before a company can be used as a subject in a research project. Read more about other approvals.
If yes, please specify	University of Stavanger and University of Malawi have been granted a permission to do research on teacher education at all the public teacher training colleges in Malawi by the Malawi Ministry of Education. For this particular college, the principal at the teacher training college will be approached and I will ask for his consent to meet the mathematics teacher educators and their students.	
12. Period for processing of personal data		
Start of project End of project	30.04.2018 30.11.2020	Start of project: The date when the sample will be contacted and/or when the collection of data will begin. End of project: The date when the data will be anonymised, deleted, or filed in order to be included in a follow-up project.
Will personal data be published (directly or indirectly)?	□ Yes, directly (name etc.) □ Yes, indirectly (identifiable background information) ■ No, anonymous	Read more about directly and indirectly identifiable personal data. NB! If personally identifiable information is to be published, explicit consent must be collected from each person, and they should be given the opportunity to read through and approve of any quotes.
What will happen to the data when the project is completed?	■ The data will be anonymised □ The data will be filed with personal identification	NB! Here we mean the data material, not the publication. Even if personal data is to be published, the remaining data is usually to be anonymised. Data which is anonymised can no longer be traced back to individuals. Read more about anonymising data.
13. Finance		
How will the project be financed?	The project is financed by Department of Educational and Sports Science at the University of Stavanger	To be filled out in the case of external financing (commissioned research etc).
14. Additional information		
Please add any additional relevant information		If the project is part of a project (or is to use data from a project) that already has been assessed by the Data Protection Official and/or has a licence from the Data Protection Authority, describe this here and state the name of the project leader, the project title and/or the project number.
15. Attachments		
Attachments	Number of attachments: 3. • interview_guide.pdf • information_letter_phd_proposal_ej_aj.pdf • questionnaire.pdf	

Page 5

Appendix 4 – Article 1

Jacinto, E. J, & Jakobsen, A. (2020). Mathematical knowledge for teaching: How do primary pre-service teachers in Malawi understand it? *African Journal of Research in Mathematics, Science and Technology Education, 24*(1), 31─40. Doi: 10.1080/18117295.2020.1735673

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Mathematical Knowledge for Teaching: How do Primary Pre-service Teachers in Malawi Understand it?

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While a consensus within the field exists that pre-service teachers need adequate knowledge and skills to become effective teachers in mathematics, less is known about how they understand the mathematical knowledge needed for teaching in elementary education. This paper explores how Malawian pre-service teachers understand the mathematical knowledge needed for teaching in primary schools. Written answers to a questionnaire survey, and data from six individual interviews collected at the beginning of the teacher-training programme will be presented. The analytical framework is based on the practice-based theory of mathematical knowledge for teaching, with a specific focus on the domains of the common, horizon and specialised content knowledge. The data revealed that, while taking theoretical courses, pre-service teaches develop only intuitive but complementary views of what is proposed by the theory, which is even the case with pre-service teachers with some prior teaching experience. Findings provide insights into the development of pre-service teachers' mathematical knowledge for teaching, and suggest that further research is needed on how pre-service teachers articulate their understanding of teaching in practice.

Keywords: *Malawian teacher training; primary school education; mathematical knowledge for teaching*

Introduction

In recent decades, some progress has been made in achieving global targets in education. However, there are significant gaps in the quality of teaching and learning in low-income countries, especially in the sub-Saharan region (Altinyelken, 2010; UNESCO, 2016). The Republic of Malawi—one of the first sub-Saharan countries to implement free primary education (Wanda & Mgomezulu, 2014)—has updated their teacher training curriculum to better prepare pre-service teachers for the challenges of teaching all subjects. In the field of mathematics, a recent study conducted at eight primary pre-service teacher colleges in Malawi showed a statistically significant increase in the pre-service teachers' mathematical knowledge for teaching during their training (Jakobsen et al., 2018). However, the overall change in mathematical knowledge for teaching was relatively small, and most pre-service teachers' mathematical knowledge for teaching showed little improvement during teacher training at their teacher-training college.

While the study of Jakobsen et al. (2018) looked at quantitative changes in pre-service teachers' mathematical knowledge for teaching during teachers' first year at teacher-training college, there is still little knowledge about what beginner teachers think and experience in their teacher-training programme. Thus, the present study aims to gain a better understanding of how pre-service teachers in Malawi understand the mathematical knowledge needed to handle the tasks of teaching in primary schools. The main question is: *How do Malawian pre-service teachers understand the mathematical knowledge for teaching in primary schools?* By mathematical knowledge for teaching, we refer to the 'mathematical knowledge that teachers need to carry out their work as teachers of mathematics' (Ball et al., 2008, p. 4), which is grounded in Shulman and Sykes's (1986) definition of the knowledge base for teaching, which they describe as 'the body of understanding and skills, and device and values, character and performance that together constitute the ability of a teacher to teach mathematics' (as cited in Fernandez, 2014, p. 82).

Framing Teachers' Knowledge within the Malawian Context

There has been a significant increase in international research focusing on teachers' learning (Castle, 2013; Cochran-Smith, 2004; Darling-Hammond & Bransford, 2005; Putnam & Borko, 1997) and on different forms of knowledge needed to prepare pre-service teachers for their future profession (Cheng et al., 2014; Rowland & Turner, 2007). In the USA and in some countries in Europe and Latin America, teacher-training programmes have in many ways advanced in these fields (Darling-Hammond, 2006; Fernández-Soria, 2013; Ries et al., 2016). These programmes have established guidelines that have contributed to the design of new educational policies, models for teaching training programmes and school curricula to assist teachers in responding to the problems arising in school. In contrast, although Asian and African countries have also presented some improvements in those areas, few studies have been conducted in these regions on pre-service teachers' learning and experiences during teacher education (Depaepe et al., 2013).

In the sub-Saharan African region, recent reform efforts have focused on teachers' mathematical knowledge as the answer to several challenges in teaching and teacher education (UNESCO, 2016). In Malawi, the Ministry of Education recently implemented a new curriculum to improve initial primary teacher education. The document specifies that the main function of teacher training colleges should be to prepare future teachers to acquire the right knowledge, skills and competencies so that they can face the challenges in this context (Malawian Institute of Education, 2017). Most of the theoretical background behind this document is based on Shulman's (1986) ideas about subject matter knowledge, pedagogical content knowledge and curricular knowledge (Malawian Institute of Education, 2017).

Although Shulman's (1986) theory outlines a knowledge base needed for effective teaching, this study relies on the practice-based theory of mathematical knowledge for teaching (MKT) developed by Ball et al. (2008). Based on Shulman's (1986) ideas, Ball et al. (2008) count on a specific conception of teaching that emphasises teachers' abilities to transform subject matter knowledge into pedagogical methods to improve students' learning (Ball & Bass, 2000). Since the focus is on shaping and refining the knowledge and skills needed to conduct teaching effectively, not only does MKT help teachers and pre-service teachers to develop decision-making skills essential for teaching certain topics in the classroom (Johnson, 2009), but it also provides a sustainable theoretical base and practical implications for teacher education programmes (Hill et al., 2005).

The theory of MKT encompasses the following six teachers' knowledge domains: common content knowledge (CCK), horizon content knowledge (HCK), specialised content knowledge (SCK), knowledge of content and students, knowledge of content and teaching and knowledge of content and curriculum. In this article, we will focus on only the domains of CCK, HCK and SCK. The first domain (CCK) refers to the mathematical knowledge commonly used or produced in a variety of settings, including outside teaching. This type of knowledge 'is not specialized understandings, but questions that typically would be answerable by others who know mathematics' (Ball et al., 2008, p. 399). Using an algorithm to find the answer for a subtraction problem is an example of CCK.

HCK is the knowledge of 'how the content being taught is situated in and connected to the broader disciplinary territory' (Jakobsen et al., 2012, p. 4642). This category involves the understanding of the subject's origins and principles, as well as how valuable it can be to students' learning. As a contribution, HCK enables teachers 'to make judgments about the importance of particular ideas or questions' of students and address 'the discipline with integrity, all resources for balancing the fundamental task of connecting learners to a vast and highly developed field' (Jakobsen et al., 2012, p. 4642).

SCK is defined as 'the mathematical knowledge unique to the work of teaching' (Ball et al., 2008, p. 400). It 'involves an uncanny kind of unpacking of mathematics that is not needed—or even desirable—in settings other than teaching' (Ball et al., 2008, p. 400). It requires knowledge beyond solid content knowledge. The ability to present mathematical ideas during instruction and to respond to students' questions is an example of the tasks that teachers perform as part of their work, which requires mathematical knowledge unique to teaching mathematics. A list of mathematical tasks of teaching are listed in Figure 1.

In the Malawian context, the list of teaching tasks presented by Ball et al. (2008) reveals similarities with the tasks of Malawian mathematics teachers. Kazima et al. (2016) found that, although the Malawian context is very different from the US context, the work of teaching seems generally similar: 'some of the tasks are more commonly recognised by teachers as applicable to the Malawian context, while other tasks are found less relevant' (Kazima et al., 2016, p. 184). A task that seems to be unique to the Malawian context as opposed to the US context is the fact that Malawian teachers regularly make use of local resources as an integral part of teaching mathematics. Owing to the lack of didactical materials, teachers use natural resources, such as stones and sticks, for teaching specific mathematical topics in elementary education, and they make their own teaching and learning aids (Kazima et al., 2016). Although Ball et al. (2008) do not draw any inferences on the ways in which the teaching tasks are carried out in Malawi, they provide essential insights about the main characteristics that underpin the work of mathematics teachers, a starting point that supports further investigation of teachers' knowledge.

Methodology and Data Analysis

To explore how Malawian pre-service teachers understand the tasks of teaching mathematics in primary schools, we conducted a *qualitative case study* (Stake, 2006) with 23 pre-service teachers at the beginning of their teacher training. All of them volunteered to answer a questionnaire survey

Presenting mathematical ideas
Responding to students' "why" questions
Finding an example to make a specific mathematical point
Recognizing what is involved in using a particular representation
Linking representations to underlying ideas and to other representations
Connecting a topic being taught to topics from prior or future years
Explaining mathematical goals and purposes to parents
Appraising and adapting the mathematical content of textbooks
Modifying tasks to be either easier or harder
Evaluating the plausibility of students' claims (often quickly)
Giving or evaluating mathematical explanations
Choosing and developing useable definitions
Using mathematical notation and language and critiquing its use
Asking productive mathematical questions
Selecting representations for particular purposes
Inspecting equivalencies

Figure 1. Mathematical tasks of teaching identified by Ball et al. (2008)

that consisted of open-ended questions about their teaching experience[1] (TE), preferences for mathematics in high school (MIHS) and interests in teaching mathematics during college (MIC). Based on the pre-service teachers' answers, six different profiles of pre-service teachers were identified. The six profiles, with the number of teachers in each of the profiles indicated, were: four pre-service teachers possessing TE, MIHS, and MIC; three having TE, but not MIHS and MIC; six with no TE, but with MIHS and MIC; four with no TE and MIHS, but with MIC; two with no TE, MIHS and MIC; and four with TE and MIC, but no MIHS. For the purpose of this paper, we considered one pre-service teacher from each of the six profiles. The list with the details of the teachers and their pseudonyms appears in Table 1.

Table 1. Selection of the research sample according to their profiles

Criteria of selection of the research sample	Pre-service teachers' anonymized names
TE/MIHS/MIC	Martin
TE/No MIHS/No MIC	Mario
No TE/MIHS/MIC	Patrick
TE/No MIHS/MIC	Clara
No TE/No MIHS/MIC	Daniel
TE/No MIHS/MIC	Denise

TE, Teaching experience; MIHS, preferences for mathematics in high school; MIC, interests in teaching mathematics during college.

Apart from the organisation and selection of the research sample, the questionnaire survey comprised a list of tasks of teaching framed by the theoretical constructs of Ball et al. (2008). The pre-service teachers ranked the level of significance of each task with the Malawian context in mind. One example from this list was the task: 'respond to children's why questions'. This specific segment of the questionnaire was a precursor for conducting individual interviews with the pre-service teachers.

In the interviews, the pre-service teachers identified and rated the significance of the tasks as important, very important or not at all important. They were also asked about their views, understanding and reasons for classifying the items on that scale. The pre-service teachers also provided insights into tasks that might be specific to the Malawian context, such as 'using natural resources in the classroom' and the type of knowledge pertaining to these tasks. This information was essential for better understanding the types of mathematical knowledge and tasks of teaching that pre-service teachers recognise as crucial for their in-field experience as a teacher as well as for their future profession.

The interviews were recorded, transcribed and analyzed using four categories, which included the significance of the mathematical knowledge used to handle the tasks of teaching in primary schools from the perspective of Malawian pre-service teachers. The four categories are: *CCK for teaching and learning, relating knowledge of out-of-curriculum content to HCK, the importance of SCK in interpreting students' errors and capacities* and *SCK: stimulating mathematics learning through different approaches*. These categories reflect the ways in which the pre-service teachers understood the tasks that might emerge in mathematics instruction, as well as the knowledge needed to tackle these tasks in the context of Malawian primary schools.

Findings and Discussion

CCK for Teaching and Learning

In this category, the pre-service teachers expressed their views and understanding about the importance of mathematical knowledge and how useful this can be for teachers and students in Malawi. The interview responses of three pre-service teachers are presented below to illustrate these views:

Daniel: The teacher should have a very good knowledge of the content before going to the classroom, so **the learners can learn what they need to learn**.

Denise: It is important for the teacher to know the content because he or she can **help the learners to understand** when they are learning mathematics.
Clara: If the teacher knows the content, he will have **more confidence in the classroom**, and **the learners will know that he is wiser**.

In the interview extracts above, the three pre-service teachers acknowledge the importance of knowing the curriculum content for the benefit of students' learning. Mastering the curriculum content seems to be a crucial factor for effective teaching and learning of mathematical content. These views reinforce the idea that teachers should be capable of teaching in several ways when possessing in-depth knowledge of curricular mathematical concepts (Ball et al., 2008). However, one pre-service teacher provided a unique view of the teachers' method: 'teaching is about helping learners to learn from simple to complex, from known to unknown; and the curriculum [content] tells us the simple concepts that they need to learn' (Martin).

Daniel, Denise and Clara all acknowledge that good knowledge of the school's curriculum content is vital to ensure good quality teaching and student learning. However, it is unclear on what basis teachers decide what is important for students. In the particular case of Martin, the knowledge of subject matter is pictured as a method to encourage students to work from a topic they know (simple) towards an unknown topic (complex) or objective. From a practical perspective, the role of teachers in this context seems to be to place value on progressive learning of the subject matter.

Relating Knowledge of Out-of-curriculum Content to HCK

This second category explores the relevance of teacher knowledge needed to teach mathematics in primary schools, from the perspective of the pre-service teachers. Although scholars recognise the differences between HCK and knowledge of content and curriculum (e.g. Jakobsen et al., 2013), in this study, the pre-service teachers still refer to knowledge about content that is not in the curriculum as a curricular horizon rather than as a mathematical horizon. This view, however, varies among the pre-service teachers. In the following passages, two pre-service teachers demonstrated a tendency to prioritise content in the curriculum rather than content outside the curriculum.

Clara: The **ministry of education already applies what the learners have to know** at the primary level, so if the teacher is out of the curriculum, he can teach things that **are not intended for primary learners.**
Martin: **Since it is not part of the learners' curriculum**, I don't think it is important.

Clara and Martin's responses tend to lessen the importance of knowledge beyond the school curriculum. In Clara's view, for instance, it is not necessary for teachers to know out-of-curriculum content since it is the governmental authorities who determine what is necessary to be taught in classrooms. In the same vein, Martin does not see teachers playing a role in deciding what to teach, nor does he acknowledge the need for knowing why something is taught.

In contrast to the pre-service teachers' insights above, two other pre-service teachers demonstrate a more balanced point of view by illustrating positive and negative aspects of the teacher knowing and using contents not included in the school curriculum to teach mathematics.

Denise: It is not **so important** to know mathematics outside the curriculum, but the teacher needs to know it, in order **to make it easier for the learners**.
Daniel: So so … **The teacher might use it to answer some questions in the classroom, but it is not so important.** He needs to focus on the contents the school suggests to teach. Each standard [grade] in Malawi has its specific content to be taught so **the teacher can learn the content from the other standards to help the learner understand it**, but it is not so important for that particular level.

Denise and Daniel present a general view of the teachers' role that teachers need to possess knowledge outside the school curriculum in order to handle the tasks of teaching, such as answering students' questions or making the content easier to learn. Daniel's understanding, in particular, seems to be explicitly linked to the knowledge needed to be used in tasks involving 'connecting a topic being taught to topics from prior or future years' (Ball et al., 2008, p. 400). This view implies that the knowledge at the mathematical horizon is relevant but only for providing links with content from higher school levels.

Patrick and Mario, in contrast, present a different view to Denise and Daniel. They suggested that teachers' knowledge should also include the ability to reflect on what they are teaching and to sequence the content of the curriculum, which is also needed for effective teaching (Ball et al., 2008).

> Patrick: It is very important because **they can reflect on what they are teaching**.
> Mario: Yes, it is important because, with other content, **you can explain better, you can mix it, and you can add new ideas**.

Patrick and Mario present a broad view of the relevance of the content outside of the curriculum, which implies that they think teachers should be open to exploring new ideas for the teaching of mathematics. Both pre-service teachers see teachers as taking an active role in mediating the school curriculum. They recognise the need for reflection on what is taught and how it can be improved. To acquire a better understanding of the content and new forms of teaching, for example, Mario believes that a teacher needs to experiment and try new ideas for teaching more effectively. This view reveals a link between the vertical and horizontal curriculum (Shulman, 1986), which are both important for mathematics instruction in primary schools. In the case of Patrick, incorporating 'complex concepts and new problems from real-life situations into the classroom may not be easy for the teacher, but it can challenge and stir creativity among the learners' (Patrick).

Patrick's assumption shows the type of knowledge that is particularly relevant and useful for teaching in Malawi: the knowledge at the mathematical horizon (Ball et al., 2008). In the literature, the concept of HCK refers to 'an awareness of how mathematical topics are related over the span of mathematics included in the curriculum' (Ball et al., 2008, p. 403). However, Patrick's idea of teaching complex concepts and incorporating problems from real-life situations into mathematics lessons seems to be a significant part of the teacher's knowledge and his/her ability to make links between curriculum topics. In this context, Kelly (1999) observes that, in some situations, curriculum content might 'limit the planning of teachers to a consideration of the content or the body of knowledge they wish to transmit or a list of the subjects to be taught or both' (p. 83). Knowledge of the content that lies outside the school curriculum can also be important to the work of teaching because it can help teachers to guide students to experience real-life situations that encompass multiple concepts (Freire, 1999).

The Importance of SCK in Interpreting Students' Errors and Capacities

Although expertise in the mathematical content both within and outside the primary school curriculum is an essential component of teaching, teachers are also faced daily with the task of evaluating the plausibility of students' claims. Any teacher who knows their students and their prior knowledge can identify, explain and predict what students can do or cannot do during learning activities (Freire, 1999). In this third category, we identified three sub-categories: thoughts of pre-service teachers concerning the limitations of identifying student capacities within their contextual settings; the role of the teacher in interpreting students' mistakes and identifying their capacities; and approaches that can be used for identifying students' problems. These insights are important to get a better picture of the pre-service teachers' understanding of the knowledge needed to explain why and how students make mistakes, and what knowledge and skills a teacher can use to minimise these mistakes. With regard to the first sub-category, three pre-service teachers spoke about the limits of knowing the students' capacities during mathematics lessons.

> Martin: Teachers should know what the learners can do, **but in Malawi, it is very difficult!** We have **so many children in the classroom** that you cannot know everyone. [...] I used to teach in four classes, each with more than one hundred students.

Patrick: Learners can come across with a different challenge or problem in their life, so the teacher **needs to be familiar with it**, or the learners **will not be involved in the lesson**.

Denise: We need **to give them a chance to interact with what we are teaching**. Thus, they can help us **create different lessons** and **improve our curriculum**.

In Martin and Patrick's comments, students' abilities seem to play a significant role in defining how a teacher should teach in Malawi. However, they also believe that contextual factors, such as the large numbers of students in the classroom, limit teachers' abilities to meet the specific needs of students. Conversely, Denise sees the understanding of students' abilities as a resource that can help teachers to make effective decisions to improve their lessons. By interacting with students' thinking, teachers can change classroom practices, especially the approachability of the content being taught.

In contrast with Martin's, Patrick's and Denise's emphasis on students' abilities, three other pre-service teachers highlight the importance of interpreting students' mistakes and dealing with them in the classroom (second sub-category). While Martin, Patrick and Denise understand that teacher knowledge also includes the knowledge of interpreting students' mistakes and identifying their capacities, Daniel, Clara and Mario explained that it further includes the ability to guide students to discover the nature of their problems themselves.

Daniel: You are there to teach, so **you are there to check the mistakes**. You should correct them so they can have **the correct information**.

Clara: You can help them by **giving another view**, so they can understand mathematics and solve the problems.

Mario: The teacher must ask them, 'How did you come up with this answer? How did you solve it?' **This way, the teacher can know where the problem comes from.** Giving them the answer will not help them understand where they are wrong.

The views above encompass the third sub-category in which the pre-service teachers acknowledge the importance of helping students to become aware of their mistakes and of new ways of solving a problem. Daniel's, Clara's and Mario's comments describe this knowledge as an important skill for primary school teachers to possess in Malawi: teachers should not only know how to identify children's inaccuracies and errors but also help them to understand why these problems occur. Such an understanding reflects a tendency for recognising and valuing different ways of helping students. It confirms that knowing how to interact with students is an important characteristic of primary school teachers (Ball et al., 2008), as it gives students an opportunity to express their thoughts and ideas and reflect on what they need to learn. From a sociocultural perspective, this view is also a valid constituent: children's learning does not occur outside a particular context or in isolation from others but through someone more familiar with the topic (i.e., a teacher) and capable of understanding and support students' learning process (Vygotsky, 1987).

SCK: Stimulating Mathematics Learning Through Different Approaches

In this fourth category, the focus is on the way in which the Malawian pre-service teachers understand the different forms of teaching mathematics apart from what the textbook suggests. This component is a significant characteristic of SCK—a type of knowledge that allows teachers to engage children in mathematical activities in the classroom (Ball et al., 2008). The data from three pre-service teachers below shows multiple ways of teaching mathematics in primary schools in Malawi.

Mario: When learners have a different situation in their life, they are facing **a new challenge** that requires **new ways to solve it**. So, teachers can use these challenges to teach mathematics.

Patrick: In Malawi, we have Kwacha. So, **you can relate it in the classroom**. How much does it cost to go to Blantyre? You must negotiate and think, you know ... Is it expensive? So, this can be a way to engage learners.

Denise: There are many different ways to teach mathematics. In Malawi, we use **place-value boxes** and **abacuses**. It is cheap and easy to manage in the classroom.

In the passage above, the three pre-service teachers illustrate different ways of thinking about multiple approaches for teaching mathematics: creating problem situations and using materials or instruments that can be manipulated for the students. With regard to such problem situations, Moura (2010) describes these as contextualised problems that allow teachers to use students' natural curiosity in order to solve problems from real-life situations. However, this type of approach should be intentionally organised to develop students' autonomy and critical thinking (Moura, 2010). In Patrick's comments, for example, we note that such an idea takes form in a common situation in Malawi: *transportation costs*. Patrick also argued that '[...] when coming to school, they [the children] have to take the minibus every day, so they can apply what they learn in class'. Therefore, contextual features appear to shape Patrick's view of SCK. Based on this finding, we can affirm that, in a Malawian setting, it seems to be important for teachers to know how to engage students in stimulating activities that challenge them, so that students can acquire concepts that emerge from and explain their reality.

Moreover, Denise presents an insightful perspective about the role of teaching tools. She understands that place-value boxes and abacuses are tools that teachers can use to help children develop a deep understanding of mathematics concepts. In this context, Miller and Stigler (1991) stress that tangible materials increase children's ability to perform mental calculations and representations, and that teaching mathematics through such materials improves the environment of the math classroom (Smith et al., 1999). Thus, the practical use of tangible materials in Malawi can also be a strong component in teachers' SCK, as it 'helps to spark students' imagination by letting them touch, move about, rearrange, and otherwise handle objects' (Kennedy, 1986, p. 9). The task of making teaching and learning more realistic and practical is also previously highlighted as a task that is more common in the Malawian context as opposed to other countries (Kazima et al., 2016).

Concluding Remarks

Scholars in educational science have outlined the importance of a distinctive brand of knowledge and skills unique to the work of teaching (Ball et al., 2008). In the framework developed by Ball et al. (2008), mathematical knowledge for teaching comprises two domains—subject matter knowledge and pedagogical content knowledge. Subject matter knowledge, in particular, has been extensively studied, as it has a significant impact on pre-service teachers' use of teaching strategies in classroom practice (Darling-Hammond & Bransford, 2005). In this paper, however, we reported on a study focusing specifically on the Malawian pre-service teachers' understanding of the knowledge related to the CCK, HCK and SCK—three domains of teachers' subject matter knowledge.

Our findings indicate that, although Malawian primary school pre-service teachers interpret the knowledge needed for teaching mathematics differently, they use solid arguments in parallel with the theoretical constructs proposed by Ball et al. (2008). By questioning the importance of the core elements that contribute to the effective teaching of mathematics (Ball et al., 2008), we observed that these pre-service teachers understand the knowledge of curricular mathematical concepts as crucial for shaping and supporting students' learning. In their view, in-depth understanding of curriculum content allows teachers to concentrate on what is essential for students to learn according to the curriculum. This ability is also seen as relevant for designing lessons that will increase student confidence as the content becomes more complex.

However, our findings also suggest that the participating pre-service teachers struggle with explaining basic characteristics of the knowledge at the mathematical horizon. Although pre-service teachers in Malawi recognise the relevance of mathematical knowledge outside the primary school curriculum, they discuss this within the sphere of the curricular, rather than the mathematical horizon. Furthermore, they regard the teacher's role in deciding what content students need to learn as limited, although they also think that teachers have ample opportunities to promote students' reflections on the nature of the mathematical concepts and their connections with different aspects of the school curriculum. These views imply a strong tendency to rely on the core curriculum as a primary source for teaching and learning at an equivalent standard, whereas the links with different school levels and real-life situations are understood as complementary sources for acquiring teaching knowledge. In sum, the participants in this study concur that teachers should utilise a range of sources, methods and tools to promote students' understanding of mathematical concepts.

The insights that have emerged from this study contribute to the understanding of the ways in which pre-service teachers understand the knowledge needed to carry out tasks of teaching mathematics in primary schools in Malawi. However, further research is needed to ascertain whether pre-service teachers' understanding of what is useful for the work of teaching corresponds with the strategies they later adopt in their teaching practice. The present study is also beneficial for the enhancement of teacher training programmes, while contributing to a better understanding of the main concepts of the practice-based theory of mathematical knowledge for teaching.

Disclosure Statement

No potential conflict of interest was reported by the authors.

Note

1. For those pre-service teachers with prior teaching experience, the experience ranged from three months up to one school year, usually as volunteers or teacher assistants in primary schools.

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Appendix 5 – Article 2

Jacinto, E. L., & Jakobsen, A. (2019). Pedagogical content knowledge for teaching mathematics: What matters for pre-service primary teachers in Malawi? In M. Graven, H. Venkat, A. Essien, & P. Vale (Eds.), *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education: Research Reports* (Vol. 2, pp.424–431). Pretoria, South Africa: PME.

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PEDAGOGICAL CONTENT KNOWLEDGE FOR TEACHING MATHEMATICS: WHAT MATTERS FOR PRESERVICE PRIMARY TEACHERS IN MALAWI?

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This paper examines how Malawian preservice teachers perceive the mathematical knowledge needed for teaching in primary schools. Drawing on the practice-based theory of mathematical knowledge for teaching, the study highlights the main elements constituting the pedagogical knowledge required for teaching mathematics in remote areas of scholarly education. Data collection instruments included a questionnaire survey and individual interviews with three entrant preservice teachers attending a teacher-training college in Malawi. The results yielded by thematic data analysis show that preservice teachers conceived components of pedagogical content knowledge in different but complementary ways to those noted in pertinent literature.

INTRODUCTION

For decades, education researchers have been exploring practices and teaching techniques to learn what can improve teachers' knowledge. They have argued that, by possessing a specific type of knowledge, a teacher is able to organize potential activities that can transform subject matter content into forms more comprehensible to students (Abell, 2007; Grossman, 1990; Hurrell, 2013; Marks, 1990; Shulman, 1987). Thus, the knowledge of how teachers think, construe, and evaluate their own teaching provides valuable evidence on what comprises and enhances the nature of such knowledge.

A particularly significant contribution to this line of research was the idea of *Pedagogical Content Knowledge* (PCK), developed by Lee Shulman (1986). In his view, PCK intersperses pedagogical and subject matter knowledge, and allows teachers to make scientific subjects meaningful and useful to learners (Shulman, 1986). This idea was further elaborated by a research team led by Debora Ball. The group made observations of preservice teachers' teaching practices and identified six domains pertinent to effective teaching of mathematical subjects (Ball, Thames, & Phelps, 2008). Those domains led to the development of the *practice-based theory of mathematical knowledge for teaching* (MKT).

Drawing on a larger study about teacher education in Malawi, this paper explores how preservice teachers perceive the elements that compose the teacher's mathematical knowledge for teaching, in particular, the PCK for teaching mathematics. Consequently, the general goal is to contribute to a new understanding of how those domains take place in remote areas of teacher education.

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TEACHERS' KNOWLEDGE DOMAINS

The particular forms of content that embody teachers' professional knowledge have become a central issue of the 21st century educational practice, raising questions whose answers can affect many educational institutions around the world. In this context, the practice-based theory of MKT comprises of the following six fundamental domains of teacher' knowledge in mathematics: *Common Content Knowledge* (CCK), *Horizon Content Knowledge* (HCK), *Specialized Content Knowledge* (SCK), *Knowledge of Content and Students* (KCS), *Knowledge of Content and Curriculum* (KCC), and *Knowledge of Content and Teaching* (KCT). Together, those domains constitute a promising conceptual framework for research on teacher knowledge (Ball, Hill, & Bass, 2005).

CCK is the first domain that incorporates the subject matter knowledge. Ball et al. (2008) described it as the type of knowledge "used in settings other than teaching" and related to circumstances typically common by others who know mathematics (p. 399). In contrast, SCK refers to a specialized knowledge unique to the work of teaching. This domain entails the mathematics knowledge and skills that a teacher needs to possess in order to create necessary conditions for students to learn the subject. SCK reflects the teachers' capacity for dealing with the teaching tasks, such as "looking for patterns in student errors" and "understanding different interpretations of the operations in ways that students need not explicitly distinguish" (Ball et al., 2008, p. 400). In addition to these domains, HCK concerns "a kind of mathematical 'peripheral vision' needed in teaching, a view of the larger mathematical landscape that teaching requires" (Ball & Bass, 2009, p. 1). HCK helps preservice teachers be aware of "how the content being taught is situated in and connected to the broader disciplinary territory" (Jakobsen, Thames, Ribeiro, & Delaney, 2012, p. 4642) and "how mathematics topics are related over the mathematics span included in the curriculum" (Ball et al., 2008, p. 403).

The second set of domains (KCT, KCS, and KCC) resides in the pedagogical content for teaching—a singular type of knowledge that allows teachers to create and be oriented in a conducive learning environment. For Freire (1996), this is the most fundamental skill, indispensable from the beginning of teaching, that helps educators, teachers, and preservice teachers to understand that "teaching is not about transferring knowledge but creating possibilities for its production or its construction" (p. 47) (Translation by the authors).

The domain of KCT addresses methodological aspects that allow teachers to understand and choose the appropriate ways to meet students' learning needs. KCT concerns lesson design and use of appropriate activities in the classroom (Herbst & Kosko, 2014). Such knowledge helps teachers to develop effective teaching practice and conceptualize their own teaching. Additionally, KCT can contribute to the future teachers' learning by addressing specific ways to organize lessons that help to them deal not only with the ongoing adaptations in the classroom environment (Goodnough, 2006) but also with problems that permeate much of our current society (Freire, 1996).

The second domain, KCS, pertains to the development of teachers' specialized content knowledge (Philipp et al., 2007). KCS mainly focuses on the students' mathematics learning and understanding, i.e., how they think about mathematics, their limits, and difficulties they experience when learning the subject. From this context, Ball et al. (2008) explained that:

> When assigning a task, teachers need to anticipate what students are likely to do with it and whether they will find it easy or hard. Teachers must also be able to hear and interpret students' emerging and incomplete thinking as expressed in the ways that pupils use language. Each of these tasks requires an interaction between specific mathematical understanding and familiarity with students and their mathematical thinking. (p. 9)

While KCS supports teachers' ability to listen to their students, KCC helps them to identify, select, and decide how a particular set of curriculum materials can benefit students' learning. This last domain, in the context of initial teacher education, appears as a potent tool for preservice teachers to explore the curriculum and instructional materials relevant for teaching (Ball et al., 2008). Possessing an effective KCC, however, implies not only knowing the curricular content that need to be followed, but also how those content areas can be better introduced in the lessons.

The introduction of these last three components by Shulman (1986) and their reconceptualization by Ball et al. (2008) has been an important step toward a better comprehension of teachers' knowledge. Moreover, as Depaepe et al. (2013) pointed out, although a significant body of research on PCK has been conducted in the US and European countries, there is evident paucity of studies focusing on the African countries. Thus, further research is necessary with the aim of providing not only new insights about the idea of PCK, but also discussing the possibilities and limits in applying those concepts in African contexts.

PRESERVICE PRIMARY TEACHER EDUCATION IN MALAWI

There are eight teacher-training colleges in Malawi. They operate with local primary schools in neighboring regions by offering a two-year program for candidates aspiring to become primary school teachers. As a part of this program, candidates review the basic topics taught in primary schools, analyze lesson models given by in-service teachers, and develop teaching activities for lower and upper grades. At the core of their training, a modular structural curriculum combines studies with a special focus on the pedagogical content knowledge for teaching and competencies for ensuring learning with understanding (Malawian Institute of Education, 2017).

Although this new curriculum appears adequate for preparing preservice teachers to teach in Malawi, a recent study conducted at the eight Malawian teacher colleges indicated that most preservice teachers begin their college program with a poor understanding of basic mathematics. Consequently, their essential knowledge and skills for teaching mathematics typically do not improve as a consequence of the teacher education (Kasoka, Jakobsen, & Kazima, 2017). These results highlight the need to better understand what beginner teachers do in their teacher-training programs in Malawi, how they understand and evaluate their own lessons, and on what bases they choose to act in particular ways instead of others.

THE RESEARCH DESIGN

The larger research study that this paper draws upon was based on a qualitative multiple-case study (Stake, 2009) with 23 primary preservice teachers selected due to the diversity in their backgrounds and teaching subject preferences. The broader study was divided into three research moments following the teacher-training calendar. The first moment consisted of asking all 23 primary preservice teachers to complete a questionnaire inquiring into their previous teaching experience and subject preferences for teaching, and individual interviews about characteristics of effective mathematics teaching in Malawi. The current investigation focuses solely on the information obtained within the first moment from three preservice teachers presenting similar traits on those criteria.

The data was organized into three units of analysis bounded by the domains of teachers' PCK (KCT, KCC, and KCS). Based on these domains, three distinct categories emerged from the data: *Decision-making in teachers' KCT*; *Relations between KCT and KCC*; and *Adaptations of the classroom activities from students' contributions and level*. These categories provided insight into the particularities of Malawian preservice teachers' perceptions of PCK for teaching mathematics.

FINDINGS

Decision-making in teachers' KCT

The passage presented in this category occurred during the interview with Martin, a preservice teacher with previous teaching experience, and a preference for teaching sciences and mathematics in primary schools. The starting point was based on the type of "knowledge needed to decide on the best examples and representations to use for given instructional objectives" (Herbst & Kosko, 2014, p. 24). This idea is perceived as a significant element in KCT, as it is vital for the work of teaching mathematics (Ball et al., 2008). The evidence that frames this notion is given below:

Researcher:	Martin, is it important for teachers to think about the design of the lesson or any strategy before starting the classes?
Martin:	Yes, it is very important!
Researcher:	Why? Don't you have a [teacher] manual? You have to follow it, right?
Martin:	Yes, we have to follow the manual! **But a good teacher also has to think about what is good or bad for the learners.** He needs to think about the lesson before, so if something happens, he or she can manage what to do. The big problem is that, in Malawi, there are so many students, that teachers cannot pay attention to everyone all the time. If they do that, they will never finish the content. **So, the teacher needs to decide when it is time to give attention to students and when it is time to move on.**
Researcher:	How does he know it is the time to move on?

Martin: **If the majority understand the topic, it is time to move on.** Learners can take revision lessons at the end of the term, so there is no problem if one or two students have problems. They will have an opportunity to review it. So this I something teachers need to consider when they are teaching.

For Martin, the knowledge about instructional practice design combines the knowledge of the organization and conduction of mathematical tasks in classrooms. Decision-making was also considered as an integral part of the primary school teacher's skill management for teaching, an idea that resembles KCT as the "knowledge of strategies and representations for teaching particular topics," as proposed by Borko and Putman (1996, p. 677). However, the context in which such skill is applied was a distinctive characteristic in Martin's view: "*The number of pupils in Malawian primary classrooms usually ranges from seventy-five to more than one hundred.*" Martin also implied that teaching under such circumstances should be based on more than following manuals; it demands organization, decision-making, and a sense of time and place for learning.

Relations between KCT and KCC

This section contains a segment of the interview by Carlos, a preservice teacher with a similar background and preferences as Martin. Carlos's responses not only describe a singular characteristic for an effective teaching in Malawi, but also show how KCC might interact with the domain of KCT from a broad curricular perspective.

Researcher: Carlos, how do you describe an effective teacher?

Carlos: Ok…, **I think an effective teacher needs to be resourcive** [resourceful]. He needs to know how to use the right [teaching] resources to engage the learners in learning a topic.

Researcher: How does he know if the [teaching] resource is good or bad for the learners?

Carlos: If the resource is too complicated, it is, of course, not a good resource.

Researcher: Can you give an example of a good resource?

Carlos: **It is hard to say because in Malawi we don't have enough material**, you know… but… if you are teaching in standard four, you can use small stones to teach multiplication by nine, but just multiplication of small numbers. Multiplication by nine is part of the curriculum in standard four, you know… But, in standard five, if you use stones to teach multiplication of big numbers, it won't be a good idea because **it will take too much time for the students to count the stones**. So, for teaching multiplication involving big numbers, **you need to think about using other sources**.

In this passage, Carlos used the word *"resourcive"* to describe the way a teacher considers effective teaching resources in the classroom. This characteristic reflects the teacher's capacity for selecting, revising, and using appropriate didactical materials to facilitate student learning. Carlos cogitated that if a teacher is capable of discerning the advantages and disadvantages of using didactical resources, he/she could create conditions that are more favorable for the learners, so that they can learn by doing. If the material is incompatible with students' learning pace, it would be for the teacher to search for proper teaching resources, explains Carlos.

Although Shulman (1987) described curriculum knowledge as a major category that includes "the grasp of the materials and programs that serve as tools of the trade" (p. 8), the knowledge and skills related to teaching materials were only recently introduced into the KCC domain (Sleep, 2009). Thus, the example provided by Carlos—*the* use of small stones to teach multiplication of small numbers by nine—depicted KCC not only as the teacher's ability to familiarize with the tools for delivering mathematical lessons, but also his/her capacity for understanding how simple objects can help a concept become a learning need for students.

Adaptations of the classroom activities from students' contributions and level

The passage below is an excerpt of the interview conducted with Clara, a preservice teacher who also acquired some teaching experience before starting her training program. She too has expressed affinity for teaching mathematics and science foundations in primary schools. The passage portrays her perceptions of focal features in KCS.

Researcher:	If a student uses a different method to solve a problem, what do you think the teacher needs to do?
Clara:	I think the teacher needs to consider it because, in Malawi, you know… students have many different methods, so the teacher must consider it.
Researcher:	But if you are teaching multiplication, for example, you know… there are many ways to teach it. How do you choose it?
Clara:	**Well… first, the learners have to learn the method of the teacher. Then, you can discuss the other methods**. If the teacher stops the class to explain every single method, it will confuse those learners who are trying to learn.
Researcher:	But how do you know which method is better?
Clara:	**Usually, the method of the teacher is easier. A good teacher will always choose the easy way for the learners.**
Researcher:	So, would you use the same method in all classrooms?
Clara:	No, **it would be different because you have to use the method according to the level of students.** In standard four, for example, you have to use methods that show why this number is like this, why this is this… **so the method also has to show the details**. In standard five, you don't need to explain the details, you can just apply it.

The transcription above reveals a significant aspect of teachers' KCS. Although students' ideas should be considered in the classroom, methods proposed by the teacher are still dominant. For Clara, the knowledge about the students' capacities can be beneficial for teachers to better understand and react to the students' ideas while teaching, but the use of multiple approaches might not be helpful for those who are still learning the content.

According to Ball et al. (2008), anticipating what students are likely to think and what they can find confusing is an essential skill that helps teachers balance and adapt their work according to students' contributions and level of learning. Moura (2010) reminds us that every idea exchanged is valuable for improving the quality of teacher's work. By knowing and interacting with their students, teachers can change their conceptions and learn new ways of acting that would incite development of learners' reasoning ability (Moura, 2010).

CONCLUSION

All three segments presented in this paper focused on PCK; more specifically, they reflected what preservice teachers in Malawi perceived as crucial for the domains of KCT, KCC, and KCS. Although each of these components assumed different forms of conceptualization, they provide a new perspective on how PCK takes place in remote areas of scholar education. Moreover, over the last decades, the concept of PCK has received different conceptualizations by the academic community, posing a challenge that many educational institutions find difficult to put in practice. These insights are relevant not only for teacher educators in Malawi, who can use them to consider their activities based on preservice teachers' specificities and needs for teaching mathematics, but also for deepening and developing the theory of MKT.

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Appendix 6 – Article 3

Article 3

Jacinto, E. J., Jakobsen, A., & Bjuland, R. (2020). Understanding of the knowledge necessary to sequence tasks in mathematical instruction: The case of Malawian pre-service teachers, *International Journal of Science and Mathematics Education*. (Accepted for revision, resubmitted)

Understanding of the Knowledge Necessary to Sequence Tasks in Mathematical Instruction: The Case of Malawian Pre-Service Teachers

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Abstract
Ability to organize tasks in a consistent manner, in line with students' needs, is a key skill pre-service teachers need to develop during teacher education. However, it is still unclear how pre-service teachers comprehend this ability and knowledge entailed in carrying out this work. This article examines how pre-service teachers develop their understanding of the knowledge necessary to sequence tasks given to students in mathematical instruction. A case study was conducted with two pre-service teachers enrolled in a two-year teacher education program in Malawi. Data was collected at two different time points (moments) in the program—at the start of their teacher education, and during their first period of teaching practice in primary schools. Data was analyzed with respect to the theoretical framing of mathematical knowledge for teaching, focusing specifically on the specialized content knowledge domain. A close examination of the gathered data revealed correspondences in the ways the two participants conceived such knowledge, as well as cognitive differences in the operationalization of this knowledge in their teaching practice. The results yielded by this study highlight the need for coherent teacher education proposals, as well as didactical strategies combining theory with practice.

Keywords: Malawian primary education, specialized content knowledge, mathematics teacher education, sequencing instructional tasks.

Introduction

Over the past several decades, significant progress has been made in achieving the targets of universal primary education, but much work remains to be done to ensure adequate supply of qualified teachers (UNESCO, 2016). In the Republic of Malawi—one of the Sub-Saharan African countries with the highest ratio of pupils per qualified teacher in primary schools (80:1) according to UNICEF (2019)—curricular reform attempts have been made to better prepare teachers and pre-service teachers for the challenges of their profession (Kazima, 2014). One significant initiative in this context has focused on ensuring that pre-service teachers acquire the knowledge, skills, and competences required to teach children effectively (Malawian Institute of Education, 2017).

Findings yielded by a recent study conducted by Jakobsen, Kazima, and Kasoka (2018), covering all teacher education colleges in Malawi, indicate that a significant change in pre-service teachers' measurable mathematical knowledge for teaching occurs as they progress through their teacher education. However, the overall growth of mathematical knowledge for teaching was small, and significant differences in the attained knowledge gains were observed across teacher education colleges. Consequently, as pointed out in our previous investigation, more research needs to be done to elucidate how this knowledge is developed and can be enhanced in Malawi (Kasoka, Jakobsen, & Kazima, 2017).

In order to respond to this call, the following research question is addressed in the present investigation: *How do Malawian pre-service teachers develop their understanding of the knowledge needed for sequencing tasks[1] for mathematical instruction*? In the sections that follow, we provide a literature review on the conceptualization of teachers' knowledge for teaching and the role of Specialized Content Knowledge (SCK) in teacher education across different countries, with specific focus on Malawi. This is followed by the description of the methodology used in the study, and an overview of the content of the lessons provided by the two participating pre-service teachers. The findings drawn from the analysis of the post-lesson interviews are then presented and discussed in relation to the research question.

Mathematical Knowledge Needed for Teaching

One of the current debates among researchers, educators, and educational stakeholders relates to the knowledge needed for teaching mathematics (Carrillo, Climent, Contreras, & Muñoz-Catalán, 2013; Fennema & Franke, 1992; Krauss, Baumert, & Blum, 2008; Ma, 1999). In addition, diverse categorizations of teacher knowledge have been developed, such as the Mathematical Knowledge *in* Teaching (Rowland, Turner, Thwaites, & Huckstep, 2005); Knowledge for Teaching (Davis & Simmt, 2006); and Mathematical Knowledge *for* Teaching (Ball, Thames, & Phelps, 2008).

When examining the mathematical knowledge for teaching, Ball et al. (2008) highlighted the need to develop three domains pertaining to a teacher's pedagogical content knowledge —Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), and Knowledge of Content and Curriculum(KCC)—and three domains related to the subject matter knowledge—Common Content Knowledge (CCK), Horizon Content Knowledge (HCK), and SCK.

SCK is a special type of knowledge that is unique to the work of teaching mathematics (Ball et al., 2008). It refers to the capability of "making features of a particular content visible and learnable by students" (Ball et al., 2008, p. 400). Such knowledge goes beyond content knowledge itself; it "addresses both mathematics substance and pedagogical appropriateness," aiming to unpack "a mathematical concept into its subcomponents to make it comprehensive for children" (Ding, 2016, p. 2). For instance, when teaching division to elementary students, a teacher must know not only different ways of interpreting the procedures students need to follow, but also develop an instructional sense of the differences between "take-away" and "comparison" models of division. In other words, "specialized content knowledge demands and also promotes common content knowledge" (Ding, 2016, p. 2).

Researchers that have sought to study mathematical knowledge required for effective teaching have found that tasks presented in the list above are often applicable across different

[1] We define tasks as the problems or activities that pre-service teachers developed and posed to students.

cultures (Cole, 2012; Fauskanger, Jakobsen, Mosvold, & Bjuland, 2012; Ng, 2012). However, Kazima, Jakobsen and Kasoka (2016) found that not all of these tasks of teaching provided by Ball et al. (2008) are relevant in the Malawian context and are thus rarely adopted by teachers in classroom instruction. On the other hand, due to the lack of didactical materials, Malawian teachers regularly use locally available resources (such as stones and sticks) to ensure that students can gradually grasp and properly learn the content (Jacinto & Jakobsen, 2020). This is in line with the Malawian pre-service teacher education program, which emphasizes the need for teachers to be able to design instructional tasks that would introduce students to progressively more difficult mathematical concepts (Malawian Institute of Education, 2017).

Despite the importance of sequencing instructional tasks for students, Ball et al. (2008) did not provide a detailed account of this aspect of teaching. Rather, the authors assumed that this is a sub-component of the following tasks of teaching: "linking representations to underlying ideas and to other representations," "connecting a topic being taught to topics from prior or future lessons," "appraising and adapting the mathematical content of textbooks," and "modifying tasks to make them either easier or harder" (Ball et al., 2008, p. 400). This shows that, even though educational researchers have not yet fully understood and defined what mathematical knowledge for teaching entails, there is a need to investigate how teachers and pre-service teachers learn and employ its constructs in different situations (Hine, 2015). This article contributes to this research stream by focusing on the development of the understanding that Malawian pre-service teachers have of the knowledge needed to *sequence instructional tasks given to foster students' learning of mathematics*. Gaining insight into the understanding pre-service teachers have of the knowledge needed to carry out related tasks will not only increase our understanding of how SCK is conceived and implemented in practice in the Malawian educational context, but will also elucidate how this domain can help to reconfigure teacher education practice and curricular demands.

Methodology

Research Design

In order to investigate how pre-service teachers in Malawi develop their understanding of the knowledge needed for sequencing tasks in mathematical instruction, a case study (Stake, 2016) involving two pre-service teachers (henceforth denoted by their pseudonyms Denise and Martin) attending a primary teacher-education college in Malawi was conducted. The college is located in the southeastern region of the country and is one of the eight institutions that prepares pre-service teachers for working in primary schools in rural areas. During their two-year course (three plus three terms), pre-service teachers must complete three components: theoretical courses at the college (terms 1 and 2); supervised teaching practice in schools (terms 3 and 4); and reflectivity activities at the college after teaching practice (terms 5 and 6).

The teacher education program follows a practitioner model of teacher education that intersperses theory and practice, as well as content and pedagogy, in teacher and learning. Its curriculum covers three learning areas denoted as Education Foundations Studies, Numeracy and Mathematics, and Literacy and Languages. The mathematics curriculum is aligned with the standards that require pre-service teachers to acquire subject matter knowledge and to develop pedagogical ways to promote students' lifelong learning (Malawian Institute of

Education, 2017). Pre-service teachers need to complete all three components to receive qualifications as primary school teachers.

Data for the present study was collected at two specific time points, defined as Initial Moments (conducted during a theoretical course in the college) and Second Moment (period when pre-service teachers are teaching in primary schools). These two data collection phases were related to the two first components of the primary teacher education program (theoretical course at the college and supervised practice). Data pertaining to the third component, reflectivity activities at the college (time when pre-service teachers return to college to take more theoretical courses and reflect on their teaching experience), was not yet available when data analysis for this study took place.

In the Initial Moment, Denise and Martin answered a questionnaire survey and took part in individual interviews. The questionnaire consisted of two sections, the first of which contained questions about favorite subject in high school, teaching experience, and preference for teaching a particular subject that developed during college. The aim of these questions was to obtain a profile of these pre-service teachers at the beginning of the course. The second section of the questionnaire included a list of types of teacher knowledge[2] adapted from the work of Ball et al. (2008). For each item on the list, the pre-service teachers were required to indicate level of significance (on a five-point Likert scale), as this helped identify the types of teaching tasks that were more/less relevant to the Malawian context. Subsequent individual interviews were guided by the questionnaire responses, as the pre-service teachers were asked to elaborate on their answers, as well as provide examples and a brief description of the motives and expectations they hold for their future career as a primary school teacher.

Data related to the Second Moment was gathered via a week-long systematic observation of Denise's and Martin's teaching practices, as well as post-lesson interviews. Since the two participating pre-service teachers were located in different schools and were teaching in a different class and standard[3], the post-interviews focused on situations that occurred during classroom instruction, resembling the teaching tasks listed in the first phase of the research. After viewing video-recordings of their own lessons, the pre-service teachers reflected and commented on the tasks they encountered in the classroom, as well as the knowledge needed to handle them effectively. The goal was to engage the pre-service teachers in a reflective analytical process that could offer better insights into their understanding of the knowledge needed to carry out tasks of teaching mathematics in primary classrooms.

Data was analyzed using *thematic analysis*, an approach that, according to Attride-Stirling (2001), allows researchers to "explore the understanding of an issue or the signification of an idea, rather than to reconcile conflicting definitions of a problem" (p. 387). The objective of thematic analysis is to analyze the patterns of meaning (themes) across a dataset or one particular aspect of phenomenon in depth (Braun & Clarke, 2012). This article reports analysis on the theme *knowledge needed to sequence tasks in mathematical*

[2] An example of these tasks and knowledge domains can be found in Jacinto & Jakobsen (2020).

[3] Primary school education in Malawi comprises eight grades, referred to as Standard 1–8.

instructional. The list of teaching tasks adopted from the Ball et al.'s (2008) framework was initially used to generate data (see Figure 1). After transcribing the recordings and identifying similarities and patterns in the data related to the two pre-service teachers, a common theme related to task sequencing emerged. This theme was later redefined for referential adequacy and was analyzed at the two previously defined time points (Initial and Second Moment).

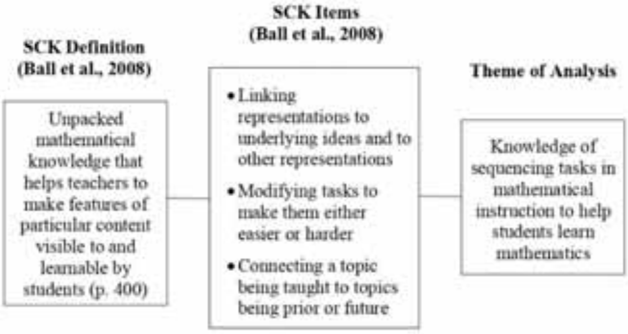


Figure 1. Theme in focus of analysis adapted from characteristics of the SCK domain proposed by Ball et al. (2008).

Participants

Denise, one of the two pre-service teachers in focus of this investigation, was 24 years old and grew up in a small village located in the central region of the country. She was the youngest sister of three, and her parents were farmers. After completing secondary school, Denise had worked as a volunteer in a primary school near her hometown, assisting a veteran teacher in organizing activities and managing children during lessons. She also had an opportunity to teach some lessons on her own when the veteran teacher was absent. In the questionnaire survey, Denise stated that she had no interest in mathematics during high school. At the college, however, she expressed preference for teaching science foundations and agriculture, two of the school subjects in the Malawi primary school curriculum. Denise stated that she wanted to become a primary school teacher so that she could be *"a model for her daughter, allowing her to see education as very valuable for our life"* (Denise, Initial Moment—Interview). Denise intended to complete her course and apply for a job in a primary school near her birthplace.

The second case study pertained to Martin—a 20-year-old man coming from the capital city of the country. His father was a secondary school teacher and his mother a housekeeper. Martin had some teaching experience prior to entering the teacher education college, and expressed interest in mathematics during high school, as well as preferences for the mathematics courses at the teacher education college. After graduation, Martin intended to obtain credentials to teach in secondary schools. He explained that being qualified to teach in both primary and secondary school "*can give many opportunities to work, so I can give my family a better life quality*" (Martin, Initial Moment—Interview). Despite notable differences in their teaching experience and subject preferences, Martin and Denise shared a similar view on the importance of the knowledge for sequencing tasks in mathematical instruction.

The following section is divided into two themes, the first of which is titled *Meaning and value of sequencing tasks for mathematical instruction: Initial understandings*, where the aim is to examine the pre-service teachers' initial views on the knowledge needed for developing a sequence of tasks used to teach a mathematical concept. As a part of the second theme—*Forms of understanding when reflecting on teaching practice*—how the pre-service teachers develop such understanding is analyzed, taking into account their own teaching experience.

Findings and Discussion

Meaning and Value of Sequencing Tasks for Mathematical Instruction: Initial Understandings

In the Initial Moment of the study, the two pre-service teachers (Denise and Martin) shared similar views on the knowledge needed for sequencing tasks for mathematical instruction, defining it as knowledge required to introduce new concepts gradually, from simple to complex. Denise specifically stated that "*the ability to make learners understand the content is one of the main characteristics of an effective teaching*" (Denise, Initial Moment—Interview). She explained that "*sometimes mathematics can be very difficult for learners, so a good teacher needs to know mathematics very well to show learners that it is not [difficult]!*" Then, she clarified this argument by saying that "*The teacher needs to teach from simple to complex, so everyone [students] can learn and use it!*" In making sense of these statements, Denise pointed out that "*teachers can help them [the learners] by giving another view, so they can learn mathematics step-by-step.*"

In a similar vein, Martin stated that "*good teachers should know how to design a lesson in a logical way, from simple to complex, as in the textbook*" (Martin, Initial Moment—Interview). During his interview, Martin elaborated on this further, referring specifically to student learning, "*teaching is about helping learners to learn from known to unknown,*" and to the teacher guidelines, "*the curriculum [the primary school curriculum] already tells us the simple concepts that they need to learn to understand the more complex ones, so the teacher does not need to do much about it.*"

The importance of appropriately sequencing tasks for mathematical instruction was clearly recognized by Denise and Martin. This demand for the work of teaching is expressed in the form of simple-to-complex approach, allowing students to gradually master the content. While Denise tended to approach this task from the students' perspective, Martin seemed to favor the school curricular guidelines. For Denise, correct sequencing of tasks given to students demands from the teachers a proper content knowledge, as well as knowledge of students' capacities. Martin, on the other hand, conceded such knowledge plays a limited role in teaching, as he claimed that the school curriculum already prescribes how teachers should introduce new concepts and examples during mathematical instruction. At this stage of the study, Denise and Martin seemed to appreciate distinct but complementary features of the knowledge entailed in the work of teaching as suggested in extant literature (Ball et al., 2008).

When examined through the lens of Ball et al.'s (2008) theory of mathematics knowledge for teaching, Martin's and Denise's views resemble some features related to SCK, for instance, "recognizing what is involved in using a particular representation," "connecting a topic being taught to topics from previous and future lessons," and "appraising and adapting the mathematical content of textbooks" (Ball et al., 2008, p. 400). Moreover, the "simple-to-complex" teaching approach, which both pre-service teachers mentioned as the knowledge unique to teaching mathematics in Malawi, seems to rely on the designing of class activities in line with students' abilities. This particular characteristic of knowledge is likely to be used to carry out teaching tasks, such as "selecting representations for particular purposes" and "evaluating the plausibility of students' claims" (Ball et al., 2008, p. 400).

It is also worth noting that both pre-service teachers expressed the idea of a simple-to-complex approach as a way to encompass a twofold goal in teaching: (1) promotion of students' confidence and ability to solve mathematical problems, and (2) alignment with the school textbook's standards. These findings provide insight into the understanding Martin and Denise have about the knowledge needed to sequence instructional tasks for mathematics teaching, which is linked to the domain of SCK in the context of Malawian primary education.

Forms of Understanding when Reflecting on Teaching Practice

This section focuses on Denise's and Martin's insights on the knowledge teachers must possess in order to appropriately sequence instructional tasks. The examples discussed here relate to practical situations observed during Denise's and Martin's lessons. The lesson excerpt analyzed to examine Denise's perspective pertains to multiplication by 3, while Martin's excerpt relates to geometrical representation of fractions. These teaching situations, given in the form of excerpts, were an important analytical source, since they were used when prompting these pre-service teachers to consider ways to improve their students' learning of mathematics.

Teaching multiplication by 3. The first excerpt pertains to Denise's instruction on multiplication by 3. She began the class by organizing her students in groups and asking them to provide the solution for the expression 2×3 she wrote on the blackboard. Then, one student from each group came to the blackboard and answered (by writing 6 after the = sign). Denise then drew two sets of three vertical lines (III III), and explained to the students that the figure represented the expression 2×3. In the sequence, Denise provided another task for the students (4×3), followed by four sets of three vertical lines (III III III III), as illustrated in Figure 2.

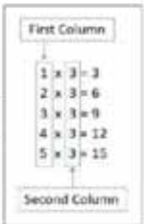
Figure 2. Representations used by Denise to explain multiplication by 3.

8

Denise's choice of representing multiplication by 3 is based on the *order of operations*. This means that the position of each number in the expression determines how such combination proceeds. In the excerpt below—taken from the post-lesson interview—Denise explicates how she interpreted this particular model of teaching multiplication.

Excerpt I: Denise's insights on the order of operations

	Interlocutor	Speech
1	Researcher:	Ok, here you are also teaching them how to multiply five by three, but it seems that you are also teaching multiplication of three by five, is that right?
2	Denise:	Yes! But multiplication of three by five is at an advanced level! In Standard 2, we start with small numbers first.
3	Researcher:	Ok, but somehow you are also teaching multiplication of numbers by five, aren't you?
4	Denise:	Yes! But there is a difference. Here, this one [first column] represents the number of groups and this one [second column] is the quantity of objects in each group.
5	Researcher:	So, the goal is …?
6	Denise:	They have to understand the meaning, you know… the numbers in that position mean what? First, they [the learners] have to find the number of the groups, and then, they have to find the number of objects in each group.
7	Researcher:	Hum … that is why you used the same structure throughout the lesson?
8	Denise:	Yes! Because if I change it, it will be different. The learners will be confused!
9	Researcher:	But they would understand, right? I mean, they should be able to solve, for example, 3 multiplied by 5?
10	Denise:	Hum … maybe! But I think it will be very difficult for them. They are not used to such expressions!



Multiplication is one of the four elementary arithmetic operations that reflects one's ability to perform repeated additions in groups (Steffe, 1994). In Denise's lesson, however, teaching of multiplication seems to include not only the idea of joining equal quantities but also a conceptual understanding of the principles that relate to each expression's factor (as indicated in Line 4 and 6 in Excerpt I). Denise acknowledged that, although both expressions 5×3 and 3×5 give the same answer (15), they can be interpreted to mean or represent two different situations: longitudinal representation in this context implies that the multiplicand symbolizes the number of groups, and the multiplier the number of units in each group. She also stated that, by changing the order of the expression, the meaning of the expression would also change, resulting in a lesson that is not designed for children in Standard 2 in Malawi (as indicated in Line 2, 8, and 10 in Excerpt I).

As can be seen from Excerpt I, multiplicand and multiplier played a substantial role in Denise's views on how multiplication by 3 should be taught in Standard 2. Denise felt that, to adopt this approach, teachers had to know the commutative property, but this was not necessary for students to learn—an assumption that implies inconsistences regarding the reasons for teachers to possess such mathematical knowledge if it is not used in practice. Nonetheless, it is worth noting that teacher's guide for primary schools in Malawi does not include commutative property as a lesson topic. Rather, teachers and pre-service teachers are advised to introduce the concept of multiplication gradually. Indeed, according to the primary school curriculum, children in Standard 2 need to learn multiplication by 2 and 3, while those in Standard 3 and 4 need to learn multiplication by 4, 5, and 6, and 7, 8, 9, and 10, respectively. In this way, children in Standard 2 learn how to calculate 7×3, for example, but will learn how to solve 3×7 in Standard 4. The principle of ordering factors according to the meaning of their positions is adopted in all primary school Standards in Malawi.

Such way of conceiving teaching of multiplication gives insight into Denise's understanding of SCK, and specifically the knowledge of sequencing tasks for multiplication instruction (Line 2) considering the students' academic level (Line 10). Teacher's awareness of students' capabilities was cited by Denise to justify the ways she chooses to make representations and explanations of the topic. As Denise's goal was not only to instruct the students on using a particular method to solve a mathematical problem, but also to teach them how multiplication works, we assumed that such type of vision, although not well-advanced, reflects how she understands SCK in a practical teaching situation. Phillip et al. (2007) also endorsed such idea by explaining that the knowledge of content and the knowledge of students can be used as a means to develop a better understanding of what constitutes SCK. This view was also shared by Thanheiser, Staples, Bartlo, Sitomer, and Heim (2010), who noted, "Connecting children's thinking and the elementary mathematics curriculum is also constituent of teachers' specialized content knowledge" (p. 12).

The next excerpt shows how Denise acknowledges such mode of teaching multiplication, as well as its effects on the students' learning. We began this particular session by asking her about the didactical resources she used for the lesson.

Excerpt II: Denise's views on the order of expressions for teaching multiplication

	Interlocutor	Speech
17	Researcher:	So, you used sticks in this lesson, right?
18	Denise:	Yes! I used sticks in this lesson!
19	Researcher:	Why sticks?
20	Denise:	Well …. For example, last week, I was teaching multiplication of numbers by two, so I was using shoes, because these objects come in twos. I was using shoes, ears, eyes, hands …. so that students can be familiar with that concept and can apply it in everyday life. I wanted to make them curious and motivate them to find objects that exist in pairs. Now, for three objects, it is more difficult, because practical examples are not locally available. So, I used stones and sticks because they are more or less similar. I asked the learners to pack them in threes, and them use those packs to solve the problems.
21	Researcher:	How about in Standard 4 and 5?
22	Denise:	In Standard 4, we teach them to count legs of tables, desks, chairs … and in Standard 5, we use, for example, fingers, toes …
23	Researcher:	I see! So, if I am going to teach this topic, do I need to know that?
24	Denise:	Yes! You must know how to model multiplication to help learners to understand the meaning. Because it is valuable for them [the learners] to know how to put them together and use it to multiply different groups of objects.
25	Researcher:	But, they will still multiply different groups of objects according to their levels? I mean in Standard 2, multiplications only involve 2 and 3, in Standard 3, they learn multiplication by 4 and 5 …
26	Denise:	Yes! According to their levels …. According to the lesson!

The procedure used to multiply numbers by 2 is the same as the one adopted when teaching multiplication by 3. In both cases, students learn how to count in groups (of two and three objects, respectively), which is, according to Denise, necessary to allow students to handle real situations where counting of equal groups of objects is required (Line 20 in Excerpt II). Her approach to teaching multiplication by 3 supported the idea that in nature objects rarely exist in groups of three (Line 20). Consequently, Denise opined that teachers should be aware of this limitation in order to help students make proper generalizations of the topic (Line 24).

According to Denise, teaching multiplication by grouping numbers is a beneficial approach for teachers, as it allows students to grasp the concepts and properties of multiplication by 3. However, by aligning the lesson with her students' level, she aimed to develop generalizations that emphasize the order of operations. In Standard 2, for instance, learners are also introduced to multiplication of two-digit numbers by 3, such as 11×3 and 12×3, but with products not exceeding 99. Therefore, the idea that learners need to know how to place objects into groups in order to multiply correctly (Line 24) meant they need to be able to count groups of numbers at the level that is determined for them by curriculum designers. Yet, despite these constraints, according to Denise, the notion of grouping helps students understand the concept of multiplication in general.

Based on these insights, we can see that Denise's views concerning the method of teaching multiplication are congruent with what she understood as an exclusive component of the work of teaching. This also aligns with the notion of SCK that allows teachers to transform content knowledge into pedagogical content knowledge (Worden, 2015). However, it is evident that, according to Denise, in order for teachers to ensure that scholarly content is understandable for students, teachers need to give precedence to certain aspects of the concept itself, rather than its properties (associative, commutative, etc.). In other words, Denise's understanding of the knowledge entailed in the teaching task aligns with the recommendations given in the teachers' guide. Nonetheless, she now provides evidence of an understanding on how abstract principles related to mathematical concepts can also benefit teaching and learning of mathematics in primary education.

Denise's case indicates that the knowledge she employs to carry out the tasks of teaching multiplication relies primarily on students' ability to develop basic generalizations, even though this overlooks certain abstract properties that could contribute to the development of complex generalizations. Thus, the notion of SCK, from the perspective of this pre-service teacher, is only a fragment of the mathematical knowledge that allows teachers to engage students in particular activities, involving different ways of representing and manipulating mathematical ideas towards the understanding of and ability to solve unusual problems, with a focus on the concepts' particularities (Hill, Ball, & Schilling, 2008).

Representing fractions in geometrical form. The analyses presented in this section illustrate how Martin understands the knowledge needed to sequence tasks for assisting students' learning of fractions. Although this can be viewed as one characteristic of a teacher's SCK domain, the primary aim is to better comprehend how Martin developed his understanding of the ways SCK can assist in the teaching of mathematics.

The example in focus of analysis occurred during a lesson on fractions—one significant topic in the primary school curriculum in Standard 4. Martin began his lesson by writing different fractions on the blackboard. Alongside each fraction, he also drew a fraction bar (or a strip diagram) in which the number of squares matched the value of its respective denominator. Martin explained: "*The goal of the lesson was to make students explore ways to represent fractions in geometrical format*" (Second Moment—Post-lesson Interview), a type of ability that helps them "*to develop a notion of parts of a whole*" (Second Moment—Post-lesson Interview). Figure 3 shows five randomly chosen students working on the blackboard to complete the fraction bars proposed by Martin.

12

Figure 3. Standard 4 students filling out strip diagrams according to their respective fractions.

As expected by Martin, the students completed the task without any problems, as *"most of them were already familiar with this type of representation."* Once this task was completed, Martin provided three similar exercises, with the exception of the last one, in which bars were perpendicular to each other (Task 3 in Figure 4).

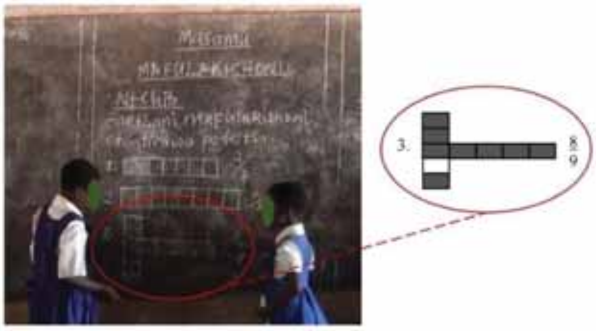


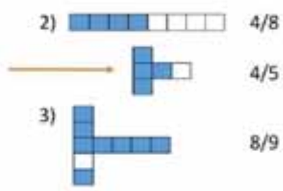
Figure 4. Students' response to Task 3 regarding representing fractions in geometrical form.

In Figure 4, the student on the left failed to complete one piece of the column in Diagram 3. As the strip diagrams in all previous figures was presented horizontally, students were required to provide almost identical answers. However, in Task 3, the horizontal–vertical combination of bars necessitated that students consider different forms of representation. The particular form of representation given by the student above was discussed in the post-lesson interview with Martin in order to explore Martin's thoughts on the response provided by this particular student.

Excerpt III: Martin's understanding of connections among tasks to assist students' learning

	Interlocutor	Speech
1	Researcher:	In this situation, why did that student choose to do that? Do you think this was a mistake?
2	Martin:	Yes! This was a mistake. She was supposed to put them all together not isolated!
3	Researcher:	But … look at the answer … is it not the same?
4	Martin:	Yes, the answer is the same, but I wanted her to put all colored strips together.
5	Researcher:	Why?
6	Martin:	Because … you know … we need to follow the textbook, otherwise, the others [students] will be confused!
7	Researcher:	But … did you tell them you wanted all colored blocks to be joined together?
8	Martin:	No, I did not! But I should have done that! It could avoid this mistake.
9	Researcher:	Ok … so, for this particular student, is there anything you could have done to help her?
10	Martin:	Yes, I could have added another example before this one, a simpler one. Like this …

Martin illustrated his idea by drawing the following figure.



11	Martin:	So, you can see that this example is much easier than the last one. So, it would help learners to understand and solve more complex problems like this one. [Martin points to the figure in Item 3 that represents the 8/9 fraction]
12	Researcher:	Hum .... Ok! But here ... in this situation, would you not consider what she did as correct?
13	Martin:	I can consider it, but sometimes it is very hard to know what the learners will do .... That's why I think it is valuable, first, to focus on the textbook.
14	Researcher:	Interesting! So, if I come to Malawi as a teacher and decide to teach this content in Standard 4, what should I know? What should I do?
15	Martin:	If you are going to teach this topic in Malawi, you must consider what learners already know and use the textbook as the main reference.

It is clear from Martin's response that he sees the student's answer as incorrect purely because it does not conform to the prescribed rules. He also highlights the importance of following the textbook to ensure that every student can develop a sense of grouping and make complex geometrical representations of fractional numbers (Line 11 in Excerpt III). In his view, *"it can be hard for them to count the bars if they don't put them together; as I said, they can get confused"* (Second Moment—Post-lesson Interview). To sustain this argument, Martin conceded that the students might have developed a better understanding of the task if he had explained to the students before the assignments that they needed fill the bars without leaving any gaps between them (Line 8 in Excerpt III). Martin also acknowledged that, as students need proper guidance, teachers must conduct mathematical lessons in a logical way, as long as this is in line with the directions provided in the textbook for that particular grade (Lines 13 and 15 in Except III).

Providing easier and connective tasks for students to solve complex mathematical problems emerged as a new level of thinking related to sequencing tasks for instruction. To better help his students to understand and represent fractions in different diagram forms, Martin suggested an extra task (Line 10 in Excerpt III) similar to the crossing-strips network proposed in Item 3 (Figure 3), but with a reduced number of bars. Such figure, according to Martin, would not only help learners to consider the individual strips conjointly, but would also mitigate possible difficulties in solving problematic situations in the classroom. *"They [learners] also have to know how to apply what they learn in the classroom in other diverse situations, not only in what is common to them,"* Martin commented.

Although the intermediate task proposed by Martin would not necessarily avoid confusion, as the figure representing the fraction 4/5 (Line 10 in Excerpt III) can also prompt students to draw bars in a crossing-strip diagram separately, it still points to aspects that should be considered when sequencing tasks for effective mathematical instruction. This way of thinking seems to be related to the idea of selecting and designing class activities (Rivas, Godino, & Castro, 2012) and modifying tasks to be both easier and harder for the students—two crucial characteristics for teaching mathematics that require SCK (Ball et al., 2008).

The design of Martin's lesson, therefore, suggests a progressive understanding based on visual representations of particular mathematical content. Once students are capable of solving tasks involving horizontal bar representations of fractions, they can start to solve more complex tasks that eventually might go beyond textbook content (Line 11 in Excerpt III). According to Martin, the knowledge of the tasks involved in such a transition is fundamental for any teacher who aims to help students to build upon knowledge they already possess (Line 15 in Excerpt III). Moreover, expanding, reducing, or modifying a lesson based on students' abilities seems to play a significant role in Martin's ways of understanding SCK in practice. Martin not only searches for ways to help students to make sense of mathematical content, but also continually assesses students' content knowledge to capitalize on it in his new proposals for mathematical tasks.

Conclusion

The work presented here provides insights into pre-service teachers' understanding of the knowledge entailed in the work of mathematics teaching. This issue is prevalent and demands attention in teacher education research. The article focused on the knowledge demands for sequencing tasks given to students in mathematical instruction, a particular component of teachers' SCK (Ball et al., 2008). Findings yielded by analyzing data pertaining to two Malawian pre-service teachers, collected at the beginning of their teacher education and during teaching practice, suggested that these pre-service teachers gradually developed their understanding of what is entailed in the work of sequencing instructional tasks, as they reflected on teaching practice. In this sense, although pre-service teachers develop different forms of understanding throughout teacher education, they appreciate the relevance of such knowledge for teachers to meet students' needs and curriculum requirements.

Several studies about the knowledge needed for teaching can be found in the literature. Still, very few authors have examined how teachers and pre-service teachers acquire and develop an understanding of such knowledge. This is particularly relevant in the Malawian context where the teacher education programs are under development. The present study provides pedagogical implications for the pre-service teachers' education in mathematics. Teacher educators could use the information provided in this article to design activities for pre-service teachers to reduce misunderstandings or discrepancies with the knowledge involved in the work of teaching. In this context, greater awareness of pre-service teachers' cognitive development could inform which strategies would facilitate acquisition of new theoretical constructs and their use in practical applications.

However, it is acknowledged that the present work is neither exhaustive nor comprehensive. Although capturing a significant portion of the problem in question, a much larger research landscape is needed to examine other types of knowledge embedded in the work of teaching mathematics. Unpacking the complexity and dynamics that encompass the teachers' learning during teacher education is a first step toward effective teacher education proposals as well as the design of valuable institutional policies for education.

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16

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Appendix 7 – Article 4

Article 4

Jacinto, E. J. (2020). An analysis of pre-service teachers' understanding of the knowledge entailed in the work of teaching: Insights from Malawi. *Journal of Mathematics Teacher Education.* (Under review)

Word count: 5553

An Analysis of Pre-service Teachers' Understanding of the Knowledge Necessary to Teach Mathematics: A Case Study in Malawi

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Abstract: Recent studies on teacher knowledge have focused on describing the knowledge needed for effective teaching, but little attention has been given on how pre-service teachers acquire and develop such knowledge as a part of teacher education programs. This gap in the extant research has motivated the present case study, focusing specifically on how pre-service teachers develop an understanding of the knowledge needed to carry out tasks of mathematics teaching. The theoretical framework underpinning the study is based on the theory of mathematical knowledge for teaching, and specialized content knowledge domain in particular. Pertinent data were gathered via a questionnaire, interviews, and a focus-group discussion with three pre-service teachers attending a two-year primary teacher education in Malawian. The analysis focused on two themes that were adapted from a set of specialized content knowledge items: (1) knowledge of instructional task progression to help students solve mathematical problems, and (2) ability to use locally available resources to create multiple concept representations. These themes included data from two different moments of pre-service teachers education (theoretical course in the college and supervised teaching practice in local schools). Findings revealed a gradual development in the three pre-service teachers' understanding of specialized content knowledge, which provides insights into the pre-service teachers' learning progression during teacher training. The study findings have implications for new methods for preparing pre-service teachers for the work of teaching in primary schools.

Keywords: Mathematics education; pre-service teacher education; teacher knowledge; specialized content knowledge.

Introduction

Quality teacher education is essential for equipping pre-service teachers with the knowledge and skills necessary for preparing the next generation to face and resolve current and anticipated societal and environmental problems and become proactive contributors to a more just, peaceful, tolerant, inclusive, secure, and sustainable world (UNESCO, 2014). In UNESCO's (2016) *Sustainable Development Agenda for Education towards 2030*, adequate supply of qualified teachers is seen as essential to accomplishing all its SDG targets[1], indicating that attention should be given to the professional training of teachers, especially those in disadvantaged areas.

Shortage of qualified teachers is a major concern in Malawi—one of the poorest countries in Sub-Saharan Africa—especially following the adoption of free primary school education to all children in 1994, which resulted in a dramatic increase in the enrolment (Wamba and Mgomezulu, 2014). Currently, the average teacher/pupil ratio is 1:77, but can reach 1:150 in schools located in rural or remote areas (UNESCO, 2019). Although the 1994 decision was a significant step toward free access to primary education in Malawi, it also created a quantity–quality trade-off, as manifested in a lack of classroom spaces, scarcity of teaching and learning materials, and insufficiency of teachers (Wamba and Mgomezulu, 2014).

In this scenario, the Malawian Initial Primary Teacher Education (IPTE) program appeared as a promising solution to improve the quality of education offered to primary pre-service teachers (Malawian Institute of Education, 2010). The pre-service teachers enrolled in the two-year IPTE program benefit from a combination of college-based education and teaching practice in primary schools. The aim of the IPTE program is to ensure that, upon graduation, pre-service teachers are "academically well-grounded and professionally competent"; "flexible and capable of adapting to the changing needs and environment of the Malawian society"; and "capable of creating and utilizing locally available resources suitable for the needs of their learners" (Malawian Institute of Education, 2017, p. 3).

The importance of combining theory and practice in the preparation of pre-service teachers is well recognised (Gravett and Ramsaroop, 2015; Korthagen, 2010), but little is known about the impacts of the IPTE program on the level of preparation of primary pre-service teachers in Malawi (Kasoka, Jakobsen, and Kazima, 2017). For example, a recent study conducted by Jakobsen, Kazima, and Kasoka (2018) involving all eight teacher training colleges (TTCs) in Malawi revealed that the pre-service teachers' level of mathematical knowledge for teaching was low, especially the knowledge needed to handle typical classroom situations. Thus, the authors called for more research not only on the IPTE curriculum, but also on the pre-service teachers' learning process during teacher training (Jakobsen et al., 2018).

The present study, as a part of a larger research project, was guided by the question: *How do Malawian primary pre-service teachers develop their understanding of the knowledge needed to carry out the tasks of teaching mathematics?* It focuses on data of three pre-service teachers during their teacher training program, which was organised into themes and analysed from the perspective of the theory of mathematical knowledge for teaching (Ball, Themes, and Phelps, 2008). The conceptual framework and related research in the field are addressed in the next section.

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[1] UNESCO's agenda contains 17 sustainable development goals (SDGs), number four of which is global access to education which includes ten sub-targets such as Target 4.1: Primary and Secondary Education, and Target 4.C: Quality of Teacher Education.

Theoretical Framework and Literature Review

Research on teachers' knowledge dates back to Shulman's investigation of pedagogical content knowledge (PCK), defined as a "knowledge base that any teacher needs for teaching a content" (Shulman, 1987, p. 4). Shulman (1986) considered it as "the most useful ways of representing and formulating the subject that make it comprehensible to others" that "includes an understanding of what makes the learning of specific topics easy or difficult" (p. 9). Since then, Ball et al. (2008) have worked to give a more comprehensive description of the mathematical knowledge for teaching.

Mathematical knowledge for teaching combines subject matter knowledge (SMK) and PCK in mathematics. PCK is divided into knowledge of the content and curriculum (KCC), knowledge of the content and students (KCS), and knowledge of content and teaching (KCT), whereas SMK comprises of common content knowledge (CCK), horizon content knowledge (HCK), and specialized content knowledge (SCK). Ball et al. (2008) described SCK as a unique component to the work of teaching, whereas CCK can be used in areas other than teaching. According to Jakobsen, Thames, Ribeiro, and Delaney (2012), SCK is distinct from HCK, which refers to "An orientation to and familiarity with the discipline (or disciplines) that contribute to the teaching of the school subject at hand, providing teachers with a sense for how the content being taught is situated in and connected to the broader disciplinary territory" (p. 4642).

Thus, SCK is the type of the knowledge and skills that make "features of a particular content visible and learnable by students" (Ball et al., 2008, p. 400). Such knowledge goes beyond content knowledge itself, as it "addresses both mathematics substance and pedagogical appropriateness," aiming to unpack "a mathematical concept into its subcomponents to make it comprehensive for children" (Ding, 2016, p. 2), as shown in Figure 1.

Presenting mathematical ideas
Responding to students' "why" questions
Finding an example to make a specific mathematical point
Recognizing what is involved in using a particular representation
Linking representations to underlying ideas and to other representations
Connecting a topic being taught to topics from prior or future years
Explaining mathematical goals and purposes to parents
Appraising and adapting the mathematical content of textbooks
Modifying tasks to be either easier or harder
Evaluating the plausibility of students' claims (often quickly)
Giving or evaluating mathematical explanations
Choosing and developing useable definitions
Using mathematical notation and language and critiquing its use
Asking productive mathematical questions
Selecting representations for particular purposes
Inspecting equivalencies

Figure 1. Mathematical tasks of teaching identified by Ball et al. (2008, p. 400).

SCK has been widely acknowledged as a crucial component to develop in teacher education (Ding, 2016). Yet, how pre-service teachers think about SCK's items they are supposed to acquire in order to teach effectively remains poorly understood (Chapman, 2015; Ferguson and Brownlee, 2018; Mosvold and Fauskanger, 2013). Addressing this gap would help determine not only the relevance of such knowledge for pre-service teachers and if they fully understand the role of theories in teaching practice (Allen and Wright, 2014; Kwenda, Adendorff, and Mosito, 2017), but also whether or not they can understand the problems and situations that happen in classrooms, why it happens, what ought to happen, and how they can overcome those problems (Chapman, 2013).

Prior research on teachers' knowledge for teaching has involved epistemic beliefs about the certainty of teaching knowledge (Ferguson and Brownlee, 2018), perceptions of teaching knowledge during teacher education (Author, 2019), construction of ideas about knowledge and teaching (Kroll, 2010), knowledge and beliefs of students' prior knowledge and the potential to develop new knowledge while solving mathematical tasks (Lee, Coomes, and Yim, 2019), changes in beliefs about the teaching and learning (Tillema, 2006), and the development of mathematical knowledge for teaching teachers (Jankvist, Clark, and Mosvold, 2020). Such a body of work has contributed with relevant insights beyond what teachers are able or not able to do in the classroom. These insights have focused on emergent issues regarding the knowledge demands entailed in the work of teaching as well as on the nature of mathematical knowledge teachers and pre-service teachers hold; how and why they hold it; how does it develop; how and when do they use it in practice; how can it be supported, changed, or enhanced to carry out tasks of teaching effectively (Chapman, 2013). Following this line of inquiry, the present study aims to contribute to this filed by ascertaining how Malawian primary pre-service teachers construct and develop their understanding of the knowledge necessary to carry out tasks of teaching in mathematics.

Methodology and Data Analysis

This qualitative case study (Stake, 2006) involved three pre-service teachers— Denise, Martin, and Mario (pseudonyms)— that enrolled in 2018 in the two years primary teacher education at one TTC in Malawi. While the participants' profiles differ, all three had a similar level of prior teaching experience. Denise developed interest for teaching mathematics and science at the TTC, Martin enjoyed learning mathematics in high school, which spurred his interest in teaching at the TTC, while Mario had never shown any interest in mathematics.

The study was conducted in three different time points (moments) of the two year teacher education (three plus three terms). The moments are labelled as Initial Moment (IM), Second Moment (SM), and Third Moment (TM). The first moment occurred at the beginning of the teacher education program that took place at the TTC (first two terms). It consisted of a questionnaire survey that included a list of open-ended questions about teaching experiences, subject preferences during high school, and teaching preferences during college. The questionnaire also contained a Likert-type scale table (Krosnick, 1999) in which pre-service teachers ranked the importance of SCK items (from extremely important to not important) in the context of Malawi. Next, the pre-service teachers took part in individual interviews, where they provided reasons and examples for their questionnaire responses.

The SM moment occurred during their supervised teaching practice (term three and four) and included observations of the pre-service teachers' teaching of lessons and post-lesson interviews. The lessons were video-recorded, and the SM post-lesson interviews focused on the teaching tasks that were identified during the pre-service teachers' teaching, and the type of knowledge used (or that could be used) to carry out those tasks effectively in the classroom. The TM phase, conducted at the end of the teacher training (term five and six), consisted of a focus group discussion (Krueger, 1998) involving all three pre-service teachers. The discussions covered both the questionnaire responses and specific episodes from the teaching experience in schools. The goal was to determine the level of pre-service teachers' understanding of teaching knowledge and its evolution over time. Table 1 synthesises all study procedures.

Table 1

Moments of Investigation and Data Collection Instruments

	Terms 1 and 2: Theoretical courses at the TTC	**Terms 3 and 4: Teaching Practice in primary schools**	**Terms 5 and 6: Students come back to TTC**
Moments	Initial Moment (IM)	Second Moment (PO)	Third Moment (TM)
Data collection instruments	Questionnaire (open questions and like-scaled quiz) and individual interviews	Systematic Observations of mathematics lessons and individual interviews after lesson	Focus Group Discussion

The data was thematically analysed (Braun and Clarke, 2012). Two themes initially emerged from the list of SCK's components, as shown in Figure 2. These themes respectively pertained to the pre-service teachers' understanding of (1) the knowledge needed to carry out instructional tasks in the classroom, and (2) multiple forms of mathematical representation. Both themes are associated with the nature of content development (Ding, 2016), a central characteristic of the SCK that relates to "topic-specific and includes knowledge about alternative ways to think about a concept and identifying mathematics present in instruction" (Chinnapan, White, and Trenholm, 2018, p. 147).

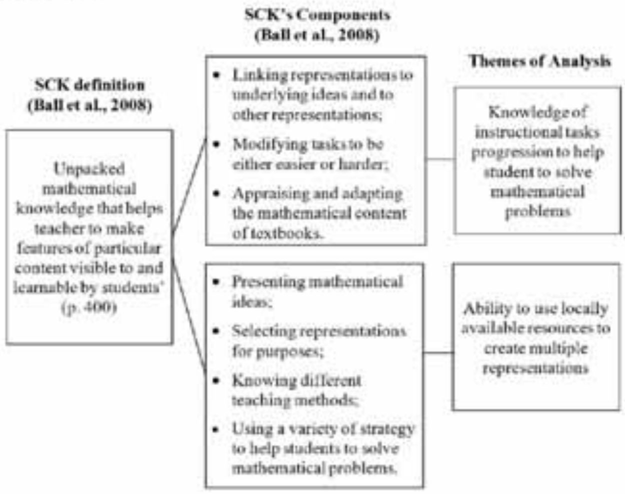

Figure 2. Themes of analysis adapted from SCK domain characteristics proposed by Ball et al. (2008).

Findings and Discussion

Data analysis reported in this section focuses on the ways pre-service teachers develop their understanding of the knowledge needed to carry out tasks that are unique to the work of teaching mathematics.

Knowledge of instructional task progression to help students solve mathematical problems

Ability to sequence instructional tasks in a manner that facilitates learning toward the lesson objective is a crucial characteristic of effective teaching (Hebert, Landin, and Solmon, 2000). Thus, this first theme related to the knowledge needed for effective progression of tasks in mathematical instruction. For Jenkins and Veal (2002), "effective task progression, or extension, requires the teacher to see the content through the student's eyes" (cited in Jenkins and Haefner, 2011, p. 48) and adjust the content to students' level of understanding.

In this theme, Denise and Martin acknowledged that tasks should be presented to the students in an easy-to-difficult/simple-to-complex sequence, and both gave examples (drawn mostly from their experiences) of how teachers can organise tasks in a progressive way to help students solve mathematical problems. However, their understanding of the knowledge needed to carry out this task differed significantly.

The case of Denise. In the IM individual interview, Denise expressed a broader view of the relevance of the knowledge needed to carry out task progression: "The teacher needs to know how to teach from simple to complex, so everyone [students] can learn and use it [the content]." However, after Denise's teaching experience, she acknowledged that "teaching mathematics from simple to complex does not mean divide the lessons into sections and that's it. It means to think about it in a way learners can understand the content gradually . . . so they can solve problems step-by-step" (Denise, SM post-lesson interview). During the TM focus group discussion, she explained that organising tasks for students in mathematical lessons should focus on "helping learners to see the steps as a whole. . . . They [students] need to understand that, if they cannot solve a problem at once, they should divide the problem into parts, so they can solve it one by one. Later, they put everything together as a whole. . . . When you give, for example, a problem like two plus three multiplied by four minus three, it can be very confusing for the learners. They can solve first two plus three, and then four minus three. Later, they can solve the multiplication and so on."

Excerpt I complements these preliminary findings and shows how Denise's understanding of the knowledge needed to sequence instructional tasks has evolved (highlighted in bold) at the TM moment. Numbers on the left column of the except helps to identify the traces of this development.

Excerpt I. Denise's comments during the TM focus group discussion.

	Interlocutor	Speech
1	Researcher:	Do you think your idea about teaching from simple-to-complex has changed since we first met?
2	Denise:	Yes, it has changed a lot! At the beginning I tried to do what my secondary teacher did, you know . . . but *now, I think it is not so easy.* It is more than using simple words. *You should think how the learner will receive the message.* Is there too much information here? How we can make easy for them to understand it? Should they learn this first? How about or this? *A good teacher must know it!*
3	Researcher:	Ok, you gave the example of two plus three multiplied by four minus three. This is supposed to be taught in a higher class, right?

8

		Why did you give this example when I asked you about teaching from simple-to-complex?
4	Denise:	Yes, that example is from a much more advanced level. We used to teach it in Standard 5 [5^{th} grade] and 6 [6^{th} grade]. . . . I used it because *teaching from simple-to-complex is a very effective. But of course it can be done in many ways*, you know . . . . For me, the teacher has to think about what is best for the students. It is not only about dividing the lessons into segments. *They should know why you are doing it. The learners should also keep in mind what is the main goal and why you are dividing it, why you are doing it step-by-step.*

Denise's responses in Excerpt 1 illustrate how her views changed as she progressed through her teacher education She stated that teachers should think how learners acquire the content (Line 2 in Excerpt I) and acknowledged that teaching from simple to complex is not a simple task (Line 2 in Excerpt I), so an effective teacher should be capable of making content gradually accessible to the students, allowing them to solve mathematical problems (Line 2 in Excerpt I). This view aligns with a direct approach to teaching that includes both instruction design but also alignment with students' needs. In this context, a teacher can break down the content into meaningful and appropriate segments to assist students' understanding of the mathematical content (Line 4 in Excerpt I). The knowledge needed to carry out this task seems to include elements of an explicit form of mathematics instruction that is intentionally and logically organised (Archer and Hughes, 2011). Such knowledge manifests as systematic expertise with the emphasis on proceeding in small steps, and checking for students' understanding, with the goal of promoting their problem-solving skills and reasoning (Line 4 in Excerpt I).

The case of Martin. Another pre-service teacher who gradually developed his understanding of task progression was Martin. In his first interview as a part of the IM, he asserted that "as the teachers' guide already tells how the content should be taught, there is no need for the teacher to do much about it. . . . It is very important to follow the guide." However, Martin's view changed as he gained practical experience teaching mathematics. In his PO post-lesson interview, Martin acknowledged, "I saw no connection between the examples in the teachers' guide, so I had to include my own examples."

Martin's decision to complement teachers' guide with his own examples marked a considerable shift in attitude. While this is a positive outcome, Zuya, Kwalat, and Attah (2016) cautioned that such practice might influence student learning negatively if the teacher has low sense of mathematics teaching self-efficacy. This negative side of instructional tasks emerged in the focus group discussion during RP, when Martin observed, "The teachers' guide is very useful, but of course it cannot predict everything in the classroom. . . . Teachers need to ask themselves: 'Will this [example] help the learners to understand the problem?' Can they [learners] see the connections?' If not, the teacher should decide what can be included to make it easier to the learners." Excerpt II complements this evidence.

Excerpt II. Martin's comments during the TM focus group discussion.

	Interlocutor	Speech
1	Martin:	When I was planning the lesson, I noticed that *some examples were too far from the others*, so learners were having problem in understanding it [the content]. *They could not see the connections between one example and others. . . .* So, I started to think *why they have no problem understanding this example, but they have problem understanding this one. It is practically the same thing, but this one is slightly different.* Then, I tried to help them with my own examples.
2	Researcher:	How do you create these examples?
3	Martin:	Well . . . I try to *look at the content as if I am learning it for the first time*, right?
4	Researcher:	Interesting!
5	Martin:	For example, if I am teaching in Standard 4, I think: *'Ok, I am a Standard 4 learner, and I already know this and this. So, what do I need to do to solve this problem?'* . . . And *if I see that the teachers' guide is not enough, I try to find other ways to help them to learn it.*
6	Researcher:	But how do you know what they need to learn?
7	Martin:	You know . . . *asking them what they already know about the content, what they find difficult.* And later, somehow, *you will discover what is important for them to succeed*!

This excerpt from the TM focus group discussion illustrates that the way Martin perceives the knowledge needed to carry out task progression evolved from IM and SM to TM. Initially, Martin believed that the teachers' guide should be strictly followed when conducting lessons in primary school classrooms. However, during teaching practice, Martin realized that the knowledge of the curriculum was insufficient to carry out teaching tasks, confirming the limitations of isolating knowledge components in teaching (Hill, Ball, and Schilling, 2008).

To carry out instructional task progression, Martin argued that the teacher needs to be able to identify and predict any problem or difficulty students might encounter during the lesson, for instance, inability to link examples provided in the teachers' guide (Line 1 in Excerpt II) and how students might recognise the similarities and differences among examples within the lessons (Lines 1 and 5, Excerpt II). Moreover, Martin opined that teachers needed to make use of the knowledge of content and students (KCS) to decide how to organise and present the tasks in the classroom (Line 3, 5, and 7, Excerpt II). Those views revealed that Martin's understanding of the knowledge needed to carry out instructional task progression at the TM was aligned with a particular SCK characteristic: "an unpacked mathematical knowledge that involves making features of particular content visible to and learnable by students" (Ball et al., 2008, p. 400).

Ability to use locally, available resources to create multiple representations

Ball et al. (2008) discussed in detail the knowledge needed to represent mathematical ideas in age-appropriate manner, which is a special knowledge component for teaching mathematics. Hudson and Miller (2006) opined that representations can be categorized as concrete, pictorial, or abstract. For instance, if a teacher uses a plastic cube to represent problems on a basic operation s(he) is utilizing concrete representation of the concept of operation. A picture of the plastic cube will be a pictorial representation while numbers or symbols will be abstract representations. Each of these categories possesses unique characteristics that contribute to the teaching and learning of mathematics.

The discussions that follow provide insights into how Mario—a pre-service teacher with experience in teaching but with no interest in mathematics—developed his understanding of the knowledge needed to teach mathematics during teacher education. He did not only acknowledge the importance of knowing about how to represent mathematics ideas but also demonstrated unique grasp of the value of understanding, creating, and using different forms of representation when teaching mathematics.

The case of Mario. In his IM's interview, Mario expressed an intuitive interpretation of the knowledge needed for mathematical representation: "It is very important for teachers to know how to introduce a concept. They must think of many ways of doing it so learners can learn the content. For example, you can use locally available resources to represent numbers, fractions, and operations. . . . In Malawi, it is a very effective way for larger classrooms as we have here. . . . When you show the learners how many forms a concept can take, they will see and use it to solve real-life problems." At this initial stage, Mario seemed to appreciate the value of mathematical representation as a way of fostering students' learning of mathematics. When asked about the relevance of such knowledge to the teacher, Martin stated: "It will make teachers better prepared to teach the topics. But you know . . . if they know different strategies, they will be able to assist as many learners as they can." This is in line with the arguments put forth by Greeno and Hall (1997).

During teaching practice, Mario's understanding of the knowledge needed for representing mathematical ideas expanded to include a new aspect—the connections between mathematical representations. An example from Mario's teaching illustrates how Mario interprets a situation from which he introduces the concept of perimeter to Standard 5 students by using different forms of representation. He first shared a story with his students about John who was asked to calculate the total length of a bent wire. As he was telling the story, Mario made use of a cardboard television he had made for this purpose with intent of capturing students' attention (see Figure 3).

Figure 3. Mario using a cardboard television to introduce the concept of perimeter.

As the story progressed, Mario actively engaged students in a discussion about John's problem. To help them find the answer, Mario drew a triangle on the blackboard with sides measuring 4, 6, and 8 cm, respectively. Below the drawing, he used these values to show how the perimeter is calculated (see Figure 4). Once the students agreed that the answer was 18 cm, Mario, using the cardboard TV, told the students that John brought the bent wire to his home, put it on the table and straightened it, so that he could measure its length. The story ended with John coming back to school and telling the teacher his findings. At the end of the lesson, Mario emphasised that the strategy for calculating the bent wire's length could be applied to other forms or objects. Then, he presented his students with a drawing of a rectangle measuring 15×6 cm, prompting

12

them to calculate its perimeter. A female student went to the board, checked the rectangle dimensions and answered 42.

Figure 4. Representations of the concept of perimeter produced by Mario.

During the post-lesson interview, when Mario was asked about his intentions in using multiple representations to introduce the concept of perimeter, he responded: "Many learners in Malawi don't know how to solve problems. I tried to make it [perimeter concept] meaningful, so they can think of the content in many ways and use it to solve problems" (Mario, SM post-lesson interview).

In Mario's initial understanding, creating opportunities for students to interpret the concept of perimeter through multiple representations seemed to play a crucial part in the teaching knowledge. This is a special component of mathematics teaching, since concepts and procedures are enhanced in teaching to help students consider not only concrete but also pictorial and abstract representations. For Mario, "As the result is the same, the teacher can help learners to see the content in different forms. In my case, I introduced perimeter in the form of a bent wire, a triangle, an expression, a straight line, and a rectangle" (Mario, SM post-lesson interview).

The experience of teaching perimeter in a Standard 5 classroom contributed to Mario's understanding of what teaching mandates in terms of knowledge. Working with multiple mathematical representations gave him useful insights into students' reasoning when solving a problem, in line with arguments put forth by Greeno and Hall (1997). Mario's understanding of the knowledge needed to use (different) mathematical representations in classroom evolved beyond not only what he was being taught at the TTC but also what the teachers' guide suggests for mathematical instruction. Locally, available resources were used by Mario as a complementary tool to introduce mathematical ideas. Seeing their practical value motivated Mario to design concrete activities through which students could develop a more robust understanding of mathematics. In this sense, Mario's knowledge of multiple representations seems to have equipped him with useful mental models of his students' appreciation of the relationship among concrete, pictorial, and abstract mathematical representations (Adu-Gyamfi and Bossé, 2014).

In the last phase of the study (TM), Mario demonstrated emergence of new insights into the knowledge needed to carry out multiple representations in the classroom. The following excerpt shows how Mario recognised such knowledge as a useful source for teaches to promote generalisation of mathematical concepts. The excerpt was extracted from one of Mario's responses to the TM focus group discussion.

Excerpt III. Mario's views during the RP focus group.

	Interlocutor	Speech
1	Researcher:	Mario, what are the benefits for a primary school teacher in knowing different forms of representing a mathematical idea?
2	Mario:	Teachers can explore *the connections between these different forms and their use in the classroom. They can think in new ways of teaching the content* . . . For example, when I was teaching the concept of perimeter in Standard 5, *I tried to create a situation that they [learners] could study the concept from different perspectives. So, if a leaner does not understand one way, he or she might understand the other.* [...] The main character, John, was asked to measure the total length of a bent wire. Later there was a triangle . . . then, he made a straight line of that triangle and then he calculated the sizes . . . *many ways of thinking about perimeter.*
3	Researcher:	So, you moved it from simple to complex, right?
4	Mario:	Yes, I gave it from concrete to abstract!
5	Researcher:	Why? You could just follow the teachers' guide, right?
6	Mario:	Yes, I could just follow the teachers' guide, but *I wanted them to understand what perimeter means and why it is important. So I used the teachers' guide and other ideas to introduce the concept of perimeter. I created it and I think it was different*!
7	Researcher:	But what about the 'proof'? You also included it in the story, and later, you showed them a different object, a rectangle.
8	Mario:	Yes, I included proof in the story because *I wanted them to make sure they have the right answer so they can avoid mistakes.* And the rectangle, I included it so *they can generalise it to other forms*, not only a triangle! In Malawi, it is very common for children to be asked about distances between places or areas of land. It somehow includes the idea of the perimeter, so it is very important for a teacher not to give the definition right way. Otherwise, he or she will confuse them.

Excerpt III indicates that, in Mario's view, the connections between different forms of mathematical representations help teachers to think about their lesson from different perspectives (Line 2 in Excerpt III). By exploring similarities and differences among representations, teachers can introduce non-standard situations to prompt students to think about the connections between different forms of the same concept (Line 2 in Excerpt III). Whilst teachers benefit in gaining knowledge about principles and relations between different forms of the subject matter, students benefit from learning different approaches that might help them better understand mathematics. For Mario, such knowledge about connections among representations was a necessary component in teaching that, in addition to knowledge of the curriculum, led to the originality of his lessons (Line 6 in Excerpt III).

Mario's understanding of the knowledge about mathematical representation also seems to be driven by the specific purposes of promoting generalisations (Line 2, 6, and 8, Excerpt III). One of these aspects is the production of meaning of the representations (Line 6 in Excerpt III). Such knowledge builds bridges from teachers' personal representations to a more conventional one (Line 2 and 8, Excerpt III), contributing to changes in beliefs about teaching (Tillema, 1998). These approaches can foster the development of generalisations of mathematical concepts, a crucial aspect when children are learning about abstract concepts (Vygotsky, 2000).

Conclusion and Implications

As SCK has been widely recognised as essential for teacher education, there is a great need to comprehend how pre-service teachers acquire and employ SCK in mathematics teaching. This article contributes to this body of literature with an examination of the understanding primary pre-service teachers in Malawi develop about the knowledge needed to carry out tasks of teaching mathematics. Two themes that emerged from the data and related SCK items were analysed in the form of knowledge of instructional task progression and ability to use locally available resources to produce multiple representations. The findings revealed that pre-service teachers developed a nuanced and well-articulated understanding of what is needed to carry out tasks constituting the work of teaching mathematics and that their understanding gradually evolved as they progressed through the teacher education program. These evolutions, however, seemed to be associated with adaptations of practical experience.

The ways pre-service teachers developed their understanding of mathematical knowledge for teaching defined their maturation and professional development during teacher education. Although each demonstrated different forms of understanding knowledge necessary to teach mathematics, these three cases revealed which elements in the process of learning how to teach can contribute to a cognitive achievement of theoretical constructs and work of teaching. Moreover, shedding light on how pre-service teachers understand the tasks of teaching and the knowledge needed to handle these tasks contributes to a better understanding of the link between theory and teacher education. Teacher educators can use this information to introduce discussions about the changing nature of the knowledge demanded for teaching, aiming to equip pre-service teachers with proper skills to ensure that they continue to learn and adapt to the rapidly changing world. Thus, more longitudinal studies are needed on the integration of theory and practice and whether theoretical constructs help pre-service teachers learn how to carry out the work of teaching more effectively. Knowing what teaching is and how it works might change the way education and teacher education should be approached in emergent contexts.

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17

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